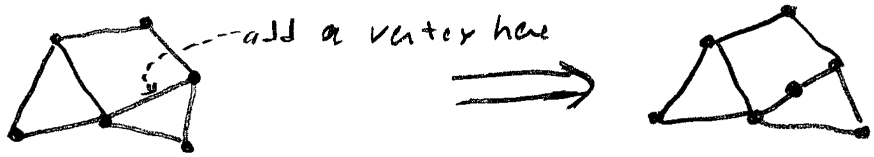
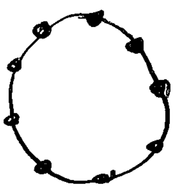
Math 110-02, Spring 2014 Sample Answers to Graded Homework #7

6.2.8 For example:



when we divide an edge and add a new vertex in the middle of it, the number of regions, F, does not change. The number of vertices, V, goes up by one, since we have added one vertex. We have replaced one edge with two edges, so the number of edges, E, also goes up by one. Since V and E each go up by one and F is unchanged, V − E + F is also unchanged.

6.2.18



9 vertices
9 edges
2 regions

$V - E + F = 9 - 9 + 2 = 2$ ✓

3 vertices
3 edges
2 regions

$V - E + F = 3 - 3 + 2 = 2$ ✓

The number of vertices is always equal to the number of edges. We can make a one-to-one correspondence between them by pairing a vertex with the edge that leaves that vertex in the counterclockwise direction.

6.2.25 We can tell which graph corresponds to which polyhedron by counting the number of regions for the graph, which must equal the number of faces of the polyhedron. The graphs come from, in order, a tetrahedron (4 sides), a cube (6), an octahedron (8), a dodecahedron (12), and an icosahedron (20).

10.4.10 A wins by plurality voting. A has two first place votes while B, C, and D only have one vote each. So A has more votes than any of the other choices.

10.4.11 B wins under "Vote for Two" voting: B has 4 votes while each of the others have 2. B also wins under Borda Count:

A: 1 + 4 + 4 + 4 + 1 = 14
B: 2 + 1 + 2 + 2 + 4 = 11
C: 3 + 2 + 1 + 3 + 3 = 12
D: 4 + 3 + 3 + 1 + 2 = 13

B has the lowest Borda count.

10.4.16 Violates the principle "Ignore the Irrelevant," since dropping a losing candidate from the race causes a different candidate to win.

Part 3

(a) B wins under plurality voting, with 10 votes to 5, 7, and 3 for the other candidates

(b) Drop the two lowest vote-getters, A and D, and hold a runoff between B and C.

B: 10

C: 5 + 7 + 3 = 15

C wins the runoff, and the election, 15 to 10

[Note: To determine which of B, C gets a group of votes, check which one is ranked higher by that group.]

(c) For IRV, drop D, the alternative with the lowest number of votes. We then have:

	5 voters	10 voters	7 voters	3 voters
1^{st} choice	A	B	C	A
2^{nd} choice	C	A	B	C
3^{rd} choice	B	C	A	B

A has the most votes, 13, which is still not a majority, so we drop C and do A vs. B:

A: 5 + 3 = 8
B: 10 + 7 = 17

B wins the election, using IRV

(d)
A: $5\times1 + 10\times3 + 7\times4 + 3\times2 = 69$
B: $5\times4 + 10\times1 + 7\times3 + 3\times4 = 63$
C: $5\times2 + 10\times4 + 7\times1 + 3\times3 = 66$
D: $5\times3 + 10\times2 + 7\times2 + 3\times1 = 52$

D wins the election using Borda Count, since it has the lowest count

(e)
A vs. B: 8 to 17
A vs C: 18 to 7
A vs D: 5 to 20
B vs C: 10 to 15
B vs D: 10 to 15
C vs D: 12 to 13

D is the Condorcet winner, since it wins against each of the other candidates in a one-on-one match.

(f) If D is eliminated, there is no Condorcet winner among A, B, C, since A beats C, C beats B, and B beats A. No alternative wins all of its matches.