(2) Suppose there is a list of infinite sequences of cards, such as

1 ⟷	$\boxed{2H}$	3D	3S	KH	KH	2D	AS	...
2 ⟷	AD	$\boxed{KS}$	3C	4C	5C	6H	JC	...
3 ⟷	KS	3D	$\boxed{2H}$	JC	JD	2H	2H	...
4 ⟷	AC	7S	7C	$\boxed{8D}$	3C	3S	3D	...
5 ⟷	9H	9S	10C	JD	$\boxed{AH}$	JD	4C	...
⋮								

We need to find a sequence of cards that is not in this list. I will make a sequence, X, containing only 2H and 3H. To get the N^{th} card in sequence ~~X~~ X, look at the N^{th} card in sequence number N. If that card is 2H, the N^{th} card in X is 3H. Otherwise, the N^{th} card in X is 2H. In the example,

X = 3H 2H 3H 2H 2H ...

X differs from sequence ~~number~~ number N in the N^{th} position. This means X is not in the list. This shows that any attempt to set up a one-to-one correspondence between the positive integers and the set of infinite sequences of cards must fail. We conclude that there are more sequences than positive integers.

(3) There are 6 outcomes on each die. For 3 dice, there are $6 \times 6 \times 6 = 216$ outcomes. There is only <u>one</u> way for the total on the dice to be 3: All dice show 1. So the probability of rolling a 3 is 1/216. There are 3 ways of rolling a 4: 112, 121, 211 —

So the probability of rolling a 4 is $3/216$.

(4) There are $2 \times 2 \times 2 \times 2 = \underline{16}$ ways for 4 coins to be tossed:

HHHH	HTHH	THHH	TTHH*
HHHT	HTHT*	THHT*	TTHT
HHTH	HTTH*	THTH*	TTTH
HHTT*	HTTT	THTT	TTTT
start with HH	start with HT	start with TH	start with TT

There are 2 outcomes where all 4 coins are the same, so the probability is $2/16$. There are 6 outcomes where the number of heads and the number of tails are the same (marked with a *), so the probability is $6/16$.

(5) If we know the 1st 4 coins come up heads, there are only two possible outcomes: HHHHH, HHHHT; the probability that the 5th coin is a head is $1/2$.
If we only know that at least 4 coins came up heads, then then the possible outcomes are HHHHT, HHHTH, HHTHH, HTHHH, THHHH, HHHHH; there are 6 possible outcomes and only 1 in which all 5 coins are heads, so the probability is $1/6$.

(6) The code is two letters <u>OR</u> a letter and a digit, so we have to <u>add</u> the possibilities for each case. The number of ways of choosing two letters is 26×26. The number of ways of choosing

a letter followed by a digit is (26×10). So the number of ways of making an order code is
$(26 \times 26) + (26 \times 10)$, or 936.

(7) $\frac{2}{54} \times \frac{1}{53}$. The probability that the 1st card is a Joker is $\frac{2}{54}$, and the probability that the second is also a Joker is then $\frac{1}{53}$. (Can also be computed as ${}^{54}C_2 / {}_2C_2$.)

The probability of getting no Joker is $\frac{52}{54} \times \frac{51}{53}$.

~~(8) Probability of 1 card without a pair is $\frac{52}{52}$. For the second card, there are~~

(8) There are 52 choices for the 1st card. For the 2nd, there are only 48 that won't make a pair with the 1st. There are then 44 that won't make a pair with either of the 1st two... The number of ways of selecting 5 cards without getting a pair is $52 \times 48 \times 44 \times 40 \times 36$. The probability of doing so is then $\frac{52 \times 48 \times 44 \times 40 \times 36}{52 \times 51 \times 50 \times 49 \times 48}$,

And the probability of at least one pair is

$$1 - \frac{52 \times 48 \times 44 \times 40 \times 36}{52 \times 51 \times 50 \times 49 \times 48} \quad [\text{about } = 0.493]$$

(9) We expect the relative frequency to be close to the actual probability. The relative frequency is $\frac{104}{1000}$. This is close to $\frac{1}{10}$, so a probability of $\frac{1}{10}$ is reasonably likely. It is quite far from $\frac{1}{5}$, so it's unlikely that the probability is $1/5$.
[For probability $\frac{1}{5}$, the event should occur about 200 times.]

(10) The order certainly matters, and repetition is not possible, so the number of ways to award 1st, 2nd, 3rd place to 8 horses is $8 \times 7 \times 6$. $({}_8P_3)$.

(11) Here, order does not matter, since we are only worried about which set of 3 horses to sell. The number of ways of choosing the 3 horses from 8 is

$$\frac{8 \times 7 \times 6}{3 \times 2 \times 1}. \quad ({}_8C_3)$$

(12) Expected value is $\frac{1}{6} \times (3-1) + \frac{2}{6} \times (2-1) + \frac{3}{6} \times (-1)$

$= \frac{2}{6} + \frac{2}{6} - \frac{3}{6} = \underline{\underline{\frac{1}{6}}}$. Since the expected value is positive, it is worth my while to play.

(13) Expected value is: $\frac{1}{6} \times (3-1) + \frac{1}{6}(2-1) + \frac{1}{6} \times (1-1) + \frac{3}{6} \times (-1)$

$= \frac{2}{6} + \frac{1}{6} + 0 - \frac{3}{6} = 0$. Since the expected value is 0, the game is "fair".

(14) The expected value is

$$\frac{1}{6}(100) + \frac{1}{6}(200) + \frac{1}{6}(300) + \frac{1}{6}(400) + \frac{1}{6}(500) + \frac{1}{6}(600)$$

$$= \frac{1}{6} \times (100 + 200 + 300 + 400 + 500 + 600)$$

$$= \frac{1}{6} \times 2100$$

$$= \underline{\underline{\$350}}$$