Your work should be submitted through Canvas by 11:59 PM on Saturday, May 9.

About the exam: This exam counts for 20% of the total grade for the course. The work that you submit for this exam should be your own. You can use course materials, including the textbook, your notes, class notes and videos from online lectures, and posted solutions to old homework and labs. You should not use other books or material from the Internet. You can ask your professor questions about the exam, but you should not receive help on the exam from other students, the math intern, your friends and family, or anyone else.

For the problems on this exam, you should not just find answers. You should present solutions. Write out your solutions carefully, including explanations to justify your work when appropriate. For example, don't just go ahead and apply L'Hôpital's rule without saying what you are doing; say "since the limit has the form $\frac{\infty}{\infty}$, we can apply L'Hôpital's rule." And if you are checking that your answer to a max/min problem is in fact a maximum, you need to say something like, "By the first derivative test..." if that is what you are using. For word problems, in particular, you should explain carefully how you got the equations that you use.

For the essay questions on this exam, you should write out clear and well-organized responses in complete sentences and paragraphs. The questions are meant to give you an opportunity to display your understanding of the central ideas from the course. Please type your answers to essay questions in a word processing application, if at all possible, and save your work as a PDF or as a Microsoft Word or Open Office document.

As usual, you can submit your work in Canvas in the form of PDFs, image files, and/or word processing documents.

---

1. Limits. (12 points.) Compute each of the following limits, using any techniques that have been covered in this course.

a) $\displaystyle\lim_{x\to+\infty} \frac{5x^4 - 3x + 7}{7x^4 + 3x^2 + 1}$ **b)** $\displaystyle\lim_{x\to 1} \frac{x^3 + 3x - 4}{2x^4 - 5x + 3}$

c) $\displaystyle\lim_{t\to 0} \frac{3\sin(t)}{\cos(3t)}$ **d)** $\displaystyle\lim_{x\to 0} \frac{\sin(x^2)}{x\sin(3x)}$

2. Derivatives. (16 points.) Compute each of the following derivatives.

a) $\displaystyle\frac{d}{dx}\left(\sin(\sqrt{x}) + \cos(x)\right)$ **b)** $\displaystyle\frac{d}{dt}\frac{\tan(t^2+1)}{\cos(1-t)}$

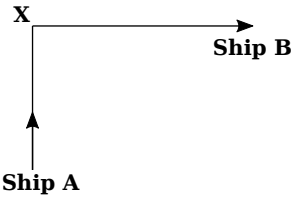
c) $\displaystyle\frac{d}{dx}\tan^{-1}(2^x)$ **d)** $\displaystyle\frac{d^2}{dx^2}\ln(e^x+1)$

3. Graphing.(8 points.) Let $f(x) = 3x^4 + 8x^3 + 6x^2 - 1$. Sketch a graph of $f(x)$ after finding the relevant information: where it is increasing, decreasing, concave up, concave down; local maxima and minima; and points of inflection. You will be graded as much on presentation as on your answers.

4. Word Problems. (24 points)

a) A cylindrical tin can with a bottom but no top is to be made using 75π square inches of tin. What is the largest possible volume for the can? (Some formulas: Volume of a cylinder is $\pi r^2 h$; area of side is $2\pi rh$; area of top or bottom is πr^2.)

b) At a certain time, a ship is 6 miles south of a point X, and is traveling north towards X at a speed of 10 miles/hour. At the same time, another ship is 8 miles east of point X and is traveling east at a speed of 7 miles/hour. How fast is the distance between the two ships changing at that time?



c) Continuing with the two ships in the previous part, assume that they continue on their current speed and direction. At what time are the two ships closest together?

d) A car is traveling along a straight road. At time $t = 0$ seconds, its velocity is 100 feet/second. Its acceleration is given by the function $a(t) = -2t$ ft/sec^2. At what time does its velocity become equal to zero, and how far has it traveled at that time (since time 0)?

5. Essay Questions. (40 points) Please type your answers in a word processing application, if possible, and submit either a PDF or a Microsoft Word or Open Office document. Remember to write your responses as well-organized essays in full sentences and paragraphs. The first two questions are worth 10 points each; the last question is worth 20.

a) Two of the important theorems about functions are the Intermediate Value Theorem and the Extreme Value Theorem. Explain what each of these theorems says and how it depends on continuity.

b) One of the major applications of derivative is optimization problems (that is, max/min problems). Explain why the first and second derivatives of a function are useful in finding maximum and minimum values of that function. (What information do they give about a function that relates to its maxima and minima?)

c) The idea of limits have been central to this course. Explain what is meant by "limit," and discuss some of the ways that limits have been used in this course. In particular, why are limits so essential to the definition of derivative (and what does this have to do with velocity and tangent lines)?