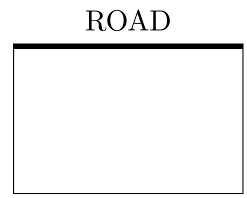
This homework on optimization problems is due on Friday, April 17.

1. A farmer wants to build a rectangular pen along the side of a road. The fence along the road has to be stronger than the fence on the other three sides of the pen. The cost for the fence along the road is \$3 per foot, while the fence for the other three sides costs \$1 per foot.



a) If the farmer has \$200 to spend on the fence, what is the maximum possible area for the pen?

b) If the area of the pen is to be 800 square feet, what is the minimum possible cost for the fence?

2. The volume of a cardboard box is to be 8 ft^3, and the base of the box is a square. Show that area of the box (including all six sides) is minimized when the box is a 2-by-2-by-2 cube. (Note that a cube gives the minimum area even if the base is not assumed to be square, but you would need multivariable calculus to show that.)

3. An athletic field is to be built. Its shape will be a rectangle with a semicircle on each end. A track around the perimeter of the field must have a length of 440 meters. What is the largest possible area for the rectangular portion of the field?

4. In class, I showed that the points $\left(\pm\sqrt{\frac{5}{2}}, \frac{5}{2}\right)$ are the points on the parabola $y = x^2$ that are closest to the point $(0,3)$. Suppose that we look at a point $(0,a)$ on the y-axis, where a can be any number. What point on the parabola is closest to $(0,a)$? The answer, of course, will depend on the value of a.

Show that for $a \geq \frac{1}{2}$, the closest points on the parabola to $(0,a)$ are $\left(\pm\sqrt{\frac{2a-1}{2}}, \frac{2a-1}{2}\right)$, and for $a \leq \frac{1}{2}$, the closest point is $(0,0)$.

5. What values of x and y will make the quantity xy^2 as large as possible, if $x \geq 0$, $y \geq 0$, and $x + y = 1$?

6. This problem generalizes the result of the previous problem. Suppose that n and m are any given positive integers, and we want to maximize the quantity $x^n y^m$ subject to the constraints that $x \geq 0$, $y \geq 0$, and $x + y = 1$. Show that the maximum is achieved when $x = \frac{n}{n+m}$ and $y = \frac{m}{n+m}$. (Hint: You will need to find the derivative a function such as $x^n(1-x)^m$. Don't try to multiply it out. Apply the product rule and the chain rule, carefully.)