Math 130, Spring 2020 Sample Answers to Homework # 11

① a) $\lim\limits_{x\to\infty} \dfrac{x^7-2x^3+8}{3x^7+2x^2+1} = \dfrac{1}{3}$ [because degrees of numerator and denominator are equal]

b) $\lim\limits_{x\to\infty} \dfrac{6x^2+1}{2x^6+1} = 0$ [because the degree of the denominator is less than the degree of the numerator.]

c) $\lim\limits_{x\to\infty} \dfrac{\pi x^3+\frac{1}{2}x^2}{ex^3-3x+\frac{13}{3}} = \dfrac{\pi}{e}$ [because degrees are equal]

② a) $\lim\limits_{x\to 0} \dfrac{\sin(x^2)}{\sin(2x)} = \lim\limits_{x\to 0} \dfrac{\cos(x^2)\cdot 2x}{\cos(2x)\cdot 2} = \dfrac{\cos(0)\cdot 0}{\cos(0)\cdot 2} = \dfrac{1\cdot 0}{1\cdot 2} = \underline{\underline{0}}$

of the form $\frac{0}{0}$

b) $\lim\limits_{x\to\infty} \dfrac{\ln(3x+1)}{\ln(2x+1)} = \lim\limits_{x\to\infty} \dfrac{\frac{1}{3x+1}\cdot 3}{\frac{1}{2x+1}\cdot 2} = \lim\limits_{x\to\infty} \dfrac{(2x+1)\cdot 3}{(3x+1)\cdot 2} = \lim\limits_{x\to\infty} \dfrac{6x+3}{6x+2} = \underline{\underline{1}}$

of the form $\frac{\infty}{\infty}$

c) $\lim\limits_{t\to 0^+} t^2 \ln(t) = \lim\limits_{t\to 0^+} \dfrac{\ln(t)}{1/t^2} = \lim\limits_{t\to 0^+} \dfrac{\frac{1}{t}}{-\frac{2}{t^3}} = \lim\limits_{t\to 0} -\frac{1}{2}t^2 = \underline{\underline{0}}$

of the form $0\cdot\infty$ of the form $\frac{\infty}{\infty}$

d) $\lim\limits_{x\to 0} \dfrac{x-\sin(x)}{x^2} = \lim\limits_{x\to 0} \dfrac{1-\cos(x)}{2x} = \lim\limits_{x\to 0} \dfrac{\sin(x)}{2} = \dfrac{0}{2} = \underline{\underline{0}}$

of the form $\frac{0}{0}$ of the form $\frac{0}{0}$

③ $\int x\sin(x)\,dx = \sin(x) - x\cos(x) + C$ is true if

$\frac{d}{dx}(\sin(x) - x\cos(x)) = x\sin(x)$;

$\frac{d}{dx}(\sin(x) - x\cos(x)) = \cos(x) - \left[x\frac{d}{dx}\cos(x) + \cos(x)\cdot\frac{d}{dx}x\right]$

$= \cos(x) - [-x\sin(x) + \cos(x)\cdot 1]$

$= \cos(x) + x\sin(x) - \cos(x)$

$= x\sin(x)$ ✓

(4) a) $\int x^3 + 3x^2 + x \, dx = \frac{x^4}{4} + 3\frac{x^3}{3} + \frac{x^2}{2} + C$

$= \frac{x^4}{4} + x^3 + \frac{x^2}{2} + C$

b) $\int \sqrt{t} + \sqrt[3]{t} \, dt = \int t^{1/2} + t^{1/3} \, dt$

$= \frac{1}{\frac{1}{2}+1} t^{1/2+1} + \frac{1}{\frac{1}{3}+1} t^{1/3+1} + C$

$= \frac{2}{3} t^{3/2} + \frac{3}{4} t^{4/3} + C$

c) $\int \sin(\theta) - \tan(\theta)\sec(\theta) \, d\theta$

$= \int \sin(\theta) d\theta - \int \tan(\theta)\sec(\theta) \, d\theta$

$= -\cos(\theta) - \tan(\theta) + C$

d) $\int \frac{x^2+1}{x} dx = \int x + \frac{1}{x} dx = \frac{x^2}{2} + \ln|x| + C$

e) $\int e^x + e^{-x} dx = e^x - e^{-x} + C$