① $\lim\limits_{x\to 4} \dfrac{x^2-3x}{x+\sqrt{x}} = \dfrac{\lim\limits_{x\to4}(x^2-3x)}{\lim\limits_{x\to4}(x+\sqrt{x})}$ by The quotient law

$= \dfrac{\lim\limits_{x\to4}(x^2) - \lim\limits_{x\to4}(3x)}{\lim\limits_{x\to4}(x) - \lim\limits_{x\to4}\sqrt{x}}$ sum and difference laws

$= \dfrac{\left(\lim\limits_{x\to4} x\right)^2 - \left(\lim\limits_{x\to4} 3\right)\left(\lim\limits_{x\to4} x\right)}{\left(\lim\limits_{x\to4} x\right) - \sqrt{\lim\limits_{x\to4} x}}$ by The power, product, and root laws

$= \dfrac{4^2 - 3\cdot 4}{4-\sqrt{4}}$ by The Two basic laws

$= \dfrac{16-12}{4-2} = \dfrac{4}{2} = 2$ by arithmetic

② a) $\lim\limits_{x\to-2} \dfrac{x^2+5x+6}{x+2} = \lim\limits_{x\to-2} \dfrac{(x+2)(x+3)}{x+2} = \lim\limits_{x\to-2}(x+3) = -2+3 = 1.$

b) $\lim\limits_{x\to-2} \dfrac{x^2+5x+4}{x+2}$. This limit is of the form $\dfrac{-2}{0}$, which means That The function will approach $+\infty$ or $-\infty$ from The left and right. $\lim\limits_{x\to-2^-} \dfrac{x^2+5x+4}{x+2} = +\infty$ because To The left of 2, numerator and denominator are both negative. $\lim\limits_{x\to-2^+} \dfrac{x^2+5x+4}{x+2} = -\infty$ because to The right of 2, The numerator is still negative but The denominator is positive. So we can only say $\lim\limits_{x\to2} \dfrac{x^2+5x+4}{x+2}$ DNE.

c) $\lim\limits_{x\to-2} \dfrac{x+2}{x^2+5x+4} = \dfrac{-2+2}{(2)^2+5(-2)+4} = \dfrac{0}{-2} = 0$

d) $\lim_{t\to 3} \frac{t^2-9}{t^2-2t-3} = \lim_{t\to 3} \frac{(t-3)(t+3)}{(t-3)(t+1)} = \lim_{t\to 3} \frac{t+3}{t+1} = \frac{3+3}{3+1} = \frac{6}{4} = \frac{3}{2}$

e) $\lim_{t\to 3} \frac{\sqrt{t+1}+4}{5-t} = \frac{\sqrt{3+1}+4}{5-3} = \frac{6}{2} = 3$

f) $\lim_{t\to 3} \frac{\sqrt{t+1}-2}{t-3} = \lim_{t\to 3} \frac{\sqrt{t+1}-2}{t-3} \cdot \frac{\sqrt{t+1}+2}{\sqrt{t+1}+2}$

$= \lim_{t\to 3} \frac{(t+1)-4}{(t-3)(\sqrt{t+1}+2)} = \lim_{t\to 3} \frac{t-3}{(t-3)(\sqrt{t+1}+2)}$

$= \lim_{t\to 3} \frac{1}{\sqrt{t+1}+2} = \frac{1}{\sqrt{3+1}+2} = \frac{1}{2+2} = \frac{1}{4}$

(3) $-1 \leq \sin(\frac{1}{x}) \leq 1$ for all $x \neq 0$. Because $x^2 > 0$ for all $x \neq 0$, we can multiply by x^2 and get $-x^2 \leq x^2 \sin(\frac{1}{x}) \leq x^2$ for all $x \neq 0$. Now, $\lim_{x\to 0}(-x^2) = \lim_{x\to 0}(x^2) = 0$. By the Squeeze Theorem, $\lim_{x\to 0} x^2 \sin(\frac{1}{x}) = 0$.

(4) a) $f(x) = \frac{x^2-1}{(x+1)(x-2)} = \frac{(x+1)(x-1)}{(x+1)(x-2)}$. There are discontinuities at $x=-1$ and at $x=2$.

Since $\lim_{x\to -1} f(x) = \lim_{x\to -1} \frac{x-1}{x-2} = \frac{-2}{-3} = \frac{2}{3}$, the discontinuity at $x=-1$ is removable.

At $x=2$, $f(x)$ is of the form $\frac{3}{0}$, so the discontinuity is infinite.

b) $g(x) = \dfrac{(x+2)(x+3)}{(x-2)(x^2-9)} = \dfrac{(x+2)(x+3)}{(x-2)(x+3)(x-3)}$.

There are discontinuities at 2, 3, and -3.

At $x=2$, $g(x)$ has the form $\frac{20}{0}$, so the discontinuity is infinite.

At $x=3$, $g(x)$ has the form $\frac{30}{0}$, so the discontinuity is infinite.

At $x=-3$, we see that $\lim\limits_{x\to -3} g(x)$ exists

(because $\lim\limits_{x\to -3} g(x) = \lim\limits_{x\to -3} \dfrac{x+2}{(x-2)(x-3)} = \dfrac{-3+2}{(-3-2)(-3-3)}$),

So the discontinuity is removable.

5 a) $p(0) = -1$ and $p(1) = 2$. Because 0 is between $p(0)$ and $p(1)$, and p is a continuous function on $[0,1]$, the IVT says there is a c in $[0,1]$ such that $p(c) = 0$. c is a root of the polynomial

b) At $x=-1$, $x^3-5x^2+2x = -1-5\cdot 1-2 = -8$.
At $x=5$, $x^3 - 5x^2+2x = 5^3-5^3+2\cdot 5 = 10$.
Since -1 is between -8 and 10, and x^3-5x^2+2x is continuous, $c^2-5c^2+2c=-1$ for some c in $[-1,5]$

c) Let $f(x) = \cos(x) - x$. $f(x)$ is continuous, $f(0) = \cos(0)-0 = 1$, $f(\pi/2) = \cos(\pi/2) - \pi/2 = -\pi/2$, and 0 is between $f(0)$ and $f(\pi/2)$. By the IVT, there is a c in $[0, \pi/2]$ such that $f(c) = 0$. That is $\cos(c) - c = 0$, or $\cos(c) = c$.