Math 130-01, Spring 2020 Sample Answers to Homework #4

① $$h'(a) = \lim_{x \to a} \frac{\sqrt{f(x)} - \sqrt{f(a)}}{x-a} = \lim_{x \to a} \frac{\sqrt{f(x)} - \sqrt{f(a)}}{x-a} \cdot \frac{\sqrt{f(x)} + \sqrt{f(a)}}{\sqrt{f(x)} + \sqrt{f(a)}}$$

$$= \lim_{x \to a} \frac{f(x) - f(a)}{(x-a) \cdot \left(\sqrt{f(x)} + \sqrt{f(a)}\right)} = \lim_{x \to a} \left(\frac{f(x) - f(a)}{x-a} \cdot \frac{1}{\sqrt{f(x)} + \sqrt{f(a)}} \right)$$

$$= \left(\lim_{x \to a} \frac{f(x) - f(a)}{x-a} \right) \left(\lim_{x \to a} \frac{1}{\sqrt{f(x)} + \sqrt{f(a)}} \right)$$

$$= f'(a) \cdot \frac{1}{\sqrt{f(a)} + \sqrt{f(a)}} = \frac{f'(a)}{2\sqrt{f(a)}}$$

② a) $$\frac{d}{dx}(2x^5 - 7x^3 + x) = 2 \cdot 5x^4 - 7 \cdot 3x^2 + 1 = 10x^4 - 21x^2 + 1$$

b) $$\frac{d}{dx}\sqrt{5x^2+1} = \frac{\frac{d}{dx}(5x^2+1)}{2\sqrt{5x^2+1}} = \frac{10x}{2\sqrt{5x^2+1}}$$

c) $$\frac{d}{dx}(5x+1)\sqrt{x^4+3} = (5x+1)\frac{d}{dx}\sqrt{x^4+3} + \sqrt{x^4+3} \cdot \frac{d}{dx}(5x+1)$$

$$= (5x+1) \frac{\frac{d}{dx}(x^4+3)}{2\sqrt{x^4+3}} + \sqrt{x^4+3} \cdot 5$$

$$= (5x+1) \cdot \frac{4x^3}{2\sqrt{x^4+3}} + \sqrt{x^4+3} \cdot 5$$

d) $$\frac{d}{dx}\left(\frac{x^2-2x}{x^3-3} \right) = \frac{(x^3-3)\frac{d}{dx}(x^2-2x) + (x^2-2x)\frac{d}{dx}(x^3-3)}{(x^3-3)^2}$$

$$= \frac{(x^3-3) \cdot (2x-2) + (x^2-2x) \cdot 3x^2}{(x-3)^2}$$

e) $\frac{d}{dx}\left(6x^3 - \frac{\sqrt{x}}{2x+1}\right) = \frac{d}{dx}6x^3 - \frac{d}{dx}\frac{\sqrt{x}}{2x+1}$

$= 18x^2 - \frac{(2x+1)\frac{d}{dx}\sqrt{x} - \sqrt{x}\frac{d}{dx}(2x+1)}{(2x+1)^2}$

$= 18x^2 - \frac{(2x+1)\cdot\frac{1}{2\sqrt{x}} - \sqrt{x}\cdot 2}{(2x+1)^2}$

③ $\frac{d}{dx} x^2 \sin(x)\cos(x) = \frac{d}{dx}(x^2)(\sin(x)\cos(x))$

$= x^2 \cdot \frac{d}{dx}(\sin(x)\cos(x)) + \sin(x)\cos(x)\cdot\frac{d}{dx}x^2$

$= x^2 \cdot \left[\sin(x)\frac{d}{dx}\cos(x) + \cos(x)\frac{d}{dx}\sin(x)\right] + \sin(x)\cos(x)\cdot 2x$

$= x^2\left(\sin(x)(-\sin(x)) + \cos(x)\cdot\cos(x)\right) + \sin(x)\cos(x)\cdot 2x$

④ a) $h'(x) = f'(x) + 2g'(x)$, so $h'(2) = 7 + 2\cdot 5 = \underline{\underline{17}}$

b) $h'(x) = f(x)g'(x) + g(x)f'(x)$

$h'(2) = 3\cdot 5 + 4\cdot 7 = 15 + 28 = \underline{\underline{43}}$

c) $h'(x) = x^3 f'(x) + f(x)\cdot 3x^2$, so $h'(2) = 2^3\cdot 7 + 3\cdot 3\cdot 2^2 = \underline{\underline{92}}$

d) $h'(x) = \frac{g(x)\cdot[x f'(x) + f(x)\cdot 1] - x f(x) g'(x)}{(g(x))^2}$,

so $h'(2) = \frac{4\cdot[2\cdot 7 + 3] - 2\cdot 3\cdot 5}{4^2} = \frac{68-30}{16} = \underline{\underline{\frac{38}{16}}}$

⑤ $v(t) = s'(t) = \underline{\underline{6t^3 - 6t + 2}}$ $a(t) = s''(t) = \underline{\underline{18t^2 - 6}}$