Math 130-01, Spring 2020 Sample Answers to Homework # 5

① a) $h(x)=f(g(x))$, $f(x)=x^7$, $g(x)=\sin(x)+1$

$f'(x)=7x^6$, $g'(x)=\cos(x)$

$$h'(x)=f'(g(x))\,g'(x)=f'(\sin(x)+1)\cos(x)=7(\sin(x)+1)^6\cos(x)$$

b) $h(x)=f(g(x))$, $f(x)=\cos(x)$, $g(x)=5\sqrt{x}$

$f'(x)=-\sin(x)$, $g'(x)=5\cdot\frac{1}{2\sqrt{x}}=\frac{5}{2\sqrt{x}}$

$$h'(x)=f'(g(x))\,g'(x)=f'(5\sqrt{x})\cdot\frac{5}{2\sqrt{x}}=-\sin(5\sqrt{x})\cdot\frac{5}{2\sqrt{x}}$$

c) $h(x)=f(g(x))$, $f(x)=\tan(x)$, $g(x)=\sin(x)+\cos(x)$

$f'(x)=\sec^2(x)$, $g'(x)=\cos(x)-\sin(x)$

$$h'(x)=f'(g(x))\,g'(x)=f'(\sin(x)+\cos(x))\cdot(\cos(x)-\sin(x))$$
$$=\sec^2(\sin(x)+\cos(x))\cdot(\cos(x)-\sin(x))$$

② a)
$$\frac{d}{dt}\left(\frac{t}{\sqrt{t^2+1}}\right)=\frac{\sqrt{t^2+1}\cdot\frac{d}{dt}t-t\frac{d}{dt}\sqrt{t^2+1}}{(\sqrt{t^2+1})^2}$$
$$=\frac{\sqrt{t^2+1}\cdot 1-t\cdot\frac{1}{2\sqrt{t^2+1}}\cdot\frac{d}{dt}(t^2+1)}{t^2+1}$$
$$=\frac{\sqrt{t^2+1}-t\cdot\frac{1}{2\sqrt{t^2+1}}\cdot 2t}{(t^2+1)^2}$$

b) $\frac{d}{dx}(3\sin(x^2)+2\cos(x^3))$
$$=3\cdot\frac{d}{dx}\sin(x^2)+2\cdot\frac{d}{dx}\cos(x^3)$$
$$=3\cdot\cos(x^2)\cdot\frac{d}{dx}x^2+2\cdot(-\sin(x^3))\cdot\frac{d}{dx}(x^3)$$
$$=3\cos(x^2)\cdot 2x+2\cdot(-\sin(x^3))\cdot 3x^2$$
$$=6x\cos(x^2)-6x^2\sin(x^3)$$

c) $\frac{d}{dz}\cos(z\sin(z)) = -\sin(z\sin(z)) \cdot \frac{d}{dz}(z\sin(z))$

$= -\sin(z\sin(z)) \cdot \left[z \cdot \frac{d}{dz}\sin(z) + \sin(z) \cdot \frac{d}{dz} z\right]$

$= -\sin(z\sin(z)) \cdot \left[z\cos(z) + \sin(z)\right]$

(3) a) $\frac{d}{dx}\left(\sqrt{\frac{4+\sin(x^4)}{5+\cos(x^5)}}\right) = \frac{1}{2\sqrt{\frac{4+\sin(x^4)}{5+\cos(x^5)}}} \cdot \frac{d}{dx}\left(\frac{4+\sin(x^4)}{5+\cos(x^5)}\right)$

$= \frac{1}{2}\sqrt{\frac{5+\cos(x^5)}{4+\sin(x^4)}} \cdot \left[\frac{(5+\cos(x^5))\frac{d}{dx}(4+\sin(x^4)) - (4+\sin(x^4))\frac{d}{dx}(5+\cos(x^5))}{(5+\cos(x^5))^2}\right]$

$= \frac{1}{2}\sqrt{\frac{5+\cos(x^5)}{4+\sin(x^4)}} \cdot \left[\frac{(5+\cos(x^5))\cos(x^4)\cdot\frac{d}{dx}x^4 - (4+\sin(x^4))\cdot(-\sin(x^5))\frac{d}{dx}x^5}{(5+\cos(x^5))^2}\right]$

$= \frac{1}{2}\sqrt{\frac{5+\cos(x^5)}{4+\sin(x^4)}} \cdot \left[\frac{(5+\cos(x^5))\cos(x^4)\cdot 4x^3 + (4+\sin(x^4))\sin(x^5)\cdot 5x^4}{(5+\cos(x^5))^2}\right]$

b) $\frac{d}{d\theta}\sin(\sin(\sin(\theta)))$

$= \cos(\sin(\sin(\theta))) \cdot \frac{d}{d\theta}\sin(\sin(\theta))$

$= \cos(\sin(\sin(\theta))) \cdot \cos(\sin(\theta)) \cdot \frac{d}{d\theta}\sin(\theta)$

$= \cos(\sin(\sin(\theta))) \cdot \cos(\sin(\theta)) \cdot \cos(\theta)$