Math 130-01, Spring 2020 Sample Answers To Homework # 6

① a) $\frac{d}{dx} e^x \sin(x) = e^x \frac{d}{dx}\sin(x) + \sin(x)\frac{d}{dx}e^x = e^x \cos(x) + \sin(x)e^x$

b) $\frac{d}{dt} \ln(2+\cos(t)) = \frac{1}{2+\cos(t)} \cdot \frac{d}{dt}(2+\cos(t)) = \frac{-\sin(t)}{2+\cos(t)}$

c) $\frac{d}{dx}\left(\frac{e^x-e^{-x}}{2}\right) = \frac{\frac{d}{dx}(e^x) - \frac{d}{dx}(e^{-x})}{2} = \frac{e^x - e^{-x}\cdot(-1)}{2} = \frac{e^x+e^{-x}}{2}$

d) $\frac{d}{ds}\left(\frac{\sin^{-1}(e^s)}{s^2}\right) = \frac{s^2 \frac{d}{ds}(\sin^{-1}(e^s)) - \sin^{-1}(e^s)\cdot \frac{d}{ds}s^2}{(s^2)^2}$

$= \frac{s^2 \cdot \frac{1}{\sqrt{1-(e^s)^2}} \cdot \frac{d}{ds}e^s - \sin^{-1}(e^s)\cdot 2s}{s^4}$

$= \frac{s^2 \cdot \frac{1}{\sqrt{1-e^{2s}}} e^s - 2s \cdot \sin^{-1}(e^s)}{s^4}$

e) $\frac{d}{dx} 4\sqrt[5]{\ln(x)+1} = 4\frac{d}{dx}(\ln(x)+1)^{1/5}$

$= 4\cdot\frac{1}{5}(\ln(x)+1)^{\frac{1}{5}-1}\cdot\frac{d}{dx}(\ln(x)+1) = \frac{4}{5}(\ln(x)+1)^{-4/5}\cdot\frac{1}{x}$

f) $\frac{d}{d\theta}\theta\cdot e^{\tan(\theta)} = \theta\cdot\frac{d}{d\theta}e^{\tan(\theta)} + e^{\tan(\theta)}\cdot\frac{d}{d\theta}\theta$

$= \theta\cdot e^{\tan(\theta)}\cdot\frac{d}{d\theta}(\tan(\theta)) + e^{\tan(\theta)}\cdot 1$

$= \theta e^{\tan(\theta)}\sec^2\theta + e^{\tan\theta}$

② If $f(x) = e^{x^2}$ Then $f'(x) = e^{x^2}\cdot\frac{d}{dx}x^2 = 2xe^{x^2}$.

The slope of The Tangent line at $x=1$ is $f'(1) = 2\cdot1\cdot e^{1^2} = 2e$

The line passes Through $(1, f(1)) = (1,e)$. Equation: $y-e = 2e(x-1)$.

③ a) If $f(x) = \ln(ax) - \ln(x)$, Then

$$f'(x) = \frac{d}{dx}(\ln(ax)) - \frac{d}{dx}(\ln(x)) =$$

$$= \frac{1}{ax} \cdot \frac{d}{dx}(ax) - \frac{1}{x} = \frac{1}{ax} \cdot a - \frac{1}{x} = \frac{1}{x} - \frac{1}{x} = 0$$

Since $f'(x) = 0$ for all $x > 0$, $f(x)$ is constant for $x > 0$

b) $f(1) = \ln(a \cdot 1) - \ln(1) = \ln(a) - 0 = \ln(a)$,

Since $f(x)$ is constant, $f(x) = \ln(a)$ for all $x > 0$.

So $\ln(ax) - \ln(x) = \ln(a)$, or $\ln(ax) = \ln(a) + \ln(x)$.

Since a was an arbitrary positive number, we have shown $\ln(ax) = \ln(a) + \ln(x)$ for all $a > 0$ and $x > 0$.

④ a) $\frac{d}{dx} x^{\sqrt{x}} = \frac{d}{dx}\left((e^{\ln(x)})^{\sqrt{x}}\right) = \frac{d}{dx} e^{\ln(x) \cdot \sqrt{x}}$

$$= e^{\ln(x) \cdot \sqrt{x}} \cdot \left[\frac{d}{dx}(\ln(x) \cdot \sqrt{x})\right]$$

$$= x^{\sqrt{x}} \cdot \left[\ln(x) \frac{d}{dx}\sqrt{x} + \sqrt{x} \ln(x)\right] = x^{\sqrt{x}} \cdot \left(\ln(x) \cdot \frac{1}{2\sqrt{x}} + \sqrt{x} \cdot \frac{1}{x}\right)$$

b) $\frac{d}{dx}(\tan(x))^{x^2} = \frac{d}{dx} e^{\ln(\tan(x)) \cdot x^2}$

$$= e^{\ln(\tan(x)) \cdot x^2} \cdot \frac{d}{dx}\left(\ln(\tan(x)) \cdot x^2\right)$$

$$= (\tan(x))^{x^2} \cdot \left(\ln(\tan(x)) \cdot \frac{d}{dx} x^2 + x^2 \frac{d}{dx} \ln(\tan(x))\right)$$

$$= (\tan(x))^{x^2} \cdot \left(\ln(\tan(x)) \cdot 2x + x^2 \cdot \frac{1}{\tan(x)} \cdot \sec^2(x)\right)$$