Math 130-01, Spring 2020 **Homework #6**

This homework is due at lab next week, on Tuesday, March 10.

1. Compute the following derivatives, using all of the rules that we have covered.

a) $\dfrac{d}{dx}\, e^x \sin(x)$ **b)** $\dfrac{d}{dt}\, \ln(2 + \cos(t))$ **c)** $\dfrac{d}{dx}\, \dfrac{e^x - e^{-x}}{2}$

d) $\dfrac{d}{ds}\, \dfrac{\sin^{-1}(e^s)}{s^2}$ **e)** $\dfrac{d}{dx}\, 4\sqrt[5]{\ln(x) + 1}$ **f)** $\dfrac{d}{d\theta}\, \theta\, e^{\tan(\theta)}$

2. Find an equation for the tangent line to the graph of $y = e^{x^2}$ at the point where $x = 1$.

3. It is true, though not so easy to prove, that if the derivative of a function $f(x)$ satisfies $f'(x) = 0$ for all x in some interval, then $f(x)$ must be constant on that interval; that is, there is a constant c such that $f(x) = c$ for all x in the interval. In this problem, you will use this fact to prove one of the laws of logarithms for the natural logarithm function $\ln(x)$.

a) Let a be a positive constant, and define $f(x) = \ln(ax) - \ln(x)$, for $x > 0$, Show that $f'(x) = 0$ for all $x > 0$. Deduce that $\ln(ax) - \ln(x)$ is a constant.

b) By plugging $x = 1$ into $\ln(ax) - \ln(x)$, deduce that the constant is $\ln(a)$, and conclude that $\ln(ax) = \ln(a) + \ln(x)$ for all positive numbers a and x.

4. Using the fact that $f(x)^{g(x)}$ can be written as $e^{g(x)\ln(f(x))}$. compute the following derivatives. (Alternatively, if you prefer the textbook's logarithmic differentiation method, you can use that technique instead.)

a) $\dfrac{d}{dx}\, x^{\sqrt{x}}$ **b)** $\dfrac{d}{dx}\, (\tan(x))^{x^2}$