This homework is due on Friday, April 17.

1. Consider a quadratic polynomial $p(x) = ax^2 + bx + c$, with $a \neq 0$. Find the critical point of $p(x)$. Use the second derivative test to decide whether the critical point is a local minimum or a local maximum for $p(x)$. The answer will depend on a, the coefficient of $p(x)$.

2. Sketch the graph of a function $f(x)$ that has all of the following properties, including the values of the function shown in the table.

- $f'(x) = 0$ for $x = -2$, -1, 0, and 2.
- $f(x)$ is increasing for $x < -2$, $-2 < x < -1$, and $0 < x < 2$.
- $f(x)$ is decreasing for $-1 < x < 0$ and $x > 2$.
- $f(x)$ has a point of inflection at $x = 1$. (It also has other inflection points.)

x	$f(x)$
-3	0
-2	1
-1	2
0	1
1	2
2	3
4	0

3. Let $g(x) = x - \sin(x)$. Find the critical points of $g(x)$. Show that $f(x)$ is always increasing, except at its critical points.

4. Consider the cubic polynomial function $f(x) = 2x^3 + 9x^2 + 12$. Find the intervals on which f is increasing and on which it is decreasing. Find any local maxima and local minima of f. Find the intervals on which f is concave up and on which it is concave down. Find any inflection points of f. Sketch the graph of $y = f(x)$. (*Note: For questions 4 though 7, you should show your work and answer all questions explicitly. Don't just draw the graph!*)

5. Consider the polynomial function $p(x) = x^4 - 2x^2$. Find the intervals on which p is increasing and on which it is decreasing. Find any local maxima and local minima of p. Find the intervals on which p is concave up and on which it is concave down. Find any inflection points of p. Sketch the graph of $y = p(x)$.

6. Consider the function $f(x) = xe^{-x}$. You can assume that $\lim_{x\to\infty} f(x) = 0$. Find the intervals on which f is increasing and on which it is decreasing. Find any local maxima and local minima of f. Find the intervals on which f is concave up and on which it is concave down. Find any inflection points of f. Sketch the graph of $y = f(x)$.

7. Consider the function $f(x) = \dfrac{x}{1+x^2}$. This function approaches 0 as x goes to $+\infty$ or to $-\infty$. Find the intervals on which f is increasing and on which it is decreasing. Find any local maxima and local minima of f. Find the intervals on which f is concave up and on which it is concave down. Find any inflection points of f. Sketch the graph of $y = f(x)$.