Welcome to the course and to the first lab of the semester! For the labs in this course, you will work in a group of 3 students (or possibly 4, if the number of students isn't divisible by 3). For this lab and in a few future labs, you will be assigned to a group randomly.

In general, labs will consist of a problem set to be turned in on Friday of the same week as the lab. A few labs will be used for review for tests, with nothing to turn in. You should not necessarily expect to finish all the problems during the lab period. Depending on how much you get done, your group might need to meet outside of class to continue work on the lab.

Your group will turn in a single lab report for the group, consisting of your group's solutions to the problems. (But if you really object to that, you can write up a lab report on your own.) Treat the write-ups as writing assignments in which you should present your solution to the problem, if you found one. Even if you did not find a solution, you can still discuss how you approached the problem and any ideas or partial solutions that you came up with. Note that you will never receive credit for unsupported answers. You should justify your work and explain your reasoning with words or calclulation or both, as appropriate.

Lab reports for this lab will be collected in class this Friday.

1. General Problem Solving. I found these problems on an old calculus lab. I don't remember where I got the idea of starting the course this way, but here goes. . . .

Some people think of mathematics as simply following rules in a mechanical way to get a correct answer. But doing real mathematics means creative problem solving. It means that you have to think up different approaches, and be willing to discard an approach and try something else when what you are doing doesn't seem to be going anywhere. It means thinking logically, but also looking for flashes of intuition. It means that sometimes you won't ever get to a solution, but when you do it can be very satisfying. Here are three problems for you to try.

a) If a clock takes 5 seconds to strike 5:00 (with 5 equally spaced chimes), how long does it take to strike 10:00 (with 10 equally spaced chimes)?

b) One of three boxes contains apples, another box contains oranges, and another box contains a mixture of apples and oranges. The boxes are labeled APPLES, ORANGES and APPLES AND ORANGES, but every label is incorrect. Can you select just one fruit from just one of the boxes and determine the correct labels?

c) Sven placed exactly in the middle among all runners in a race. Dan was slower than Sven, in 10th place, and Lars was in 16th place. How many runners were in the race?

2. Graphs and Lines. We will not review precalculus at the start of the class. I will review some things from Chapter 1 of the textbook as we need them, but one thing that is particularly important from the start is an ability to work with lines, equations of lines, and graphs of functions. Here are a few problems to try, for review.

a) Recall that the *slope* of the line through two points, (x_1, y_1) and (x_2, y_2) is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$, and that the line has an equation of the form $y = mx + b$. Find an equation for the line that passes through the points $(3, 5)$ and $(-2, 8)$.

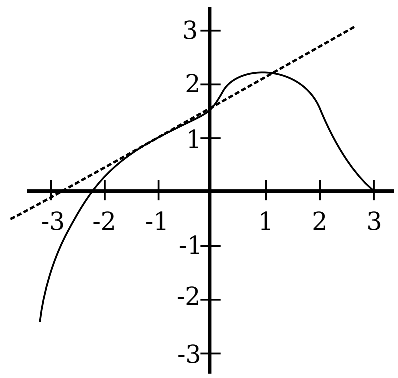
b) Are the three points $(6, 4)$, $(-1, 2)$, and $(12, 7)$ collinear? (That is, are they all exactly on one straight line?) Don't forget to justify your answer and explain your reasoning!

c) Suppose that f is the function given by $f(x) = 3x^2 + 5x + 1$. Is the point $(10, 349)$ on the graph of this function? (Why or why not?)

d) Still letting $f(x) = 3x^2 + 5x + 1$, find all points on the graph that have y-coordinate 3.

e) Still letting $f(x) = 3x^2 + 5x + 1$, find an equation for the line through the points $(1, f(1))$ and $(3, f(3))$.

f) An important concept in calculus is the *tangent line*, a line that follows a curve as closely as possible at one point. The following picture shows the graph of a function and a tangent line to that graph at the point $(-1, 1)$. Based on the picture, what is the slope of the tangent line? You can only get an estimate for the slope. Discuss how you got your estimate and how accurate you think your estimate is.



g) Now, using the same graph, find an estimate for the slope of the tangent line to the graph at the point where $x = 2$. Again, discuss how you got your estimate and how accurate you think your estimate is.

3. A Paradox. The Greek philosopher Zeno didn't believe that motion is possible. He thought that any kind of change is impossible. To prove this, he came up with several famous "paradoxes." Calculus is supposed to have solved these paradoxes and to have made motion possible again. You might have heard of Zeno's paradox of Achilles and the tortoise. Here is another of his paradoxes. You should discuss it and write up your reaction or response (hopefully more than "that's just silly").

Think of an arrow in flight. Look at it at any given moment of time. (Think of taking an instantaneous snapshot.) In that one moment of time, it's not moving—it has no time to move during a single instant of time! But if it's not moving at any given instant of time, it's clear that it doesn't move at all. If it doesn't move at any instant of time, when could it move?

4. An infinite series of events. You drop a ball from a height of four feet. Let's say that this ball always bounces back to exactly half of the height from which it falls. So, after the first bounce, it rises to a height of two feet, and then falls back to the ground and bounces again. After the second bounce it rises to a height of one foot. After the third bounce, to a height of 1/2 foot. And after the fourth, to a height of 1/4 foot.

a) Assuming that the ball really does bounce an infinite number of times, how far does it travel in total? It's possible to get an exact number, and once you do you will know something about the idea of a "limit."

b) Given that the ball bounces an infinite number of times, do you think that it bounces forever? This is something you might not be able to answer definitively, so maybe you can think about these questions: Is it plausible that the ball bounces an infinite number of times but does **not** bounce forever? Is it plausible that it bounces an infinite number of times but **does** bounce forever?