Math 130-01, Spring 2020 Sample Answers to Lab #2

① a) $\displaystyle \lim_{x\to 2} \frac{x^2-x-2}{x^2+x-6}$

$\displaystyle = \lim_{x\to 2} \frac{(x-2)(x+1)}{(x-2)(x+3)}$ algebra

$\displaystyle = \lim_{x\to 2} \frac{x+1}{x+3}$ algebra

$\displaystyle = \frac{\lim_{x\to 2}(x+1)}{\lim_{x\to 2}(x+3)}$ quotient law

$\displaystyle = \frac{\lim_{x\to 2} x + \lim_{x\to 2} 1}{\lim_{x\to 2} x + \lim_{x\to 3} 3}$ sum law

$\displaystyle = \frac{2+1}{2+3}$ basic laws

$\displaystyle = \frac{3}{5}$ arithmetic

b) $\displaystyle \lim_{x\to -1} \frac{x^2+2x+1}{x^2+x}$

$\displaystyle = \lim_{x\to -1} \frac{(x+1)^2}{x(x+1)}$ algebra

$\displaystyle = \lim_{x\to -1} \frac{x+1}{x}$ algebra

$\displaystyle = \frac{\lim_{x\to -1}(x+1)}{\lim_{x\to -1} x}$ quotient law

$\displaystyle = \frac{\left(\lim_{x\to -1} x\right) + \left(\lim_{x\to -1} 1\right)}{\lim_{x\to -1} x}$ sum law

$\displaystyle = \frac{-1+1}{-1}$ basic laws

$\displaystyle = \frac{0}{-1} = 0$ arithmetic

c) $\displaystyle \lim_{x\to 1} \left(\frac{x-1}{3x^2-4x+1}\right)^3$

$\displaystyle = \left(\lim_{x\to 1} \frac{x-1}{3x^2-4x+1}\right)^3$ power law

$\displaystyle = \left(\lim_{x\to 1} \frac{x-1}{(x-1)(3x-1)}\right)^3$ algebra

$\displaystyle = \left(\lim_{x\to 1} \frac{1}{3x-1}\right)^3$ algebra

$\displaystyle = \left(\frac{\lim_{x\to 1} 1}{\lim_{x\to 1}(3x-1)}\right)^3$ quotient law

$\displaystyle = \left(\frac{\lim_{x\to 1} 1}{\left(\lim_{x\to 1} 3x\right) - \left(\lim_{x\to 1} 1\right)}\right)^3$ difference law

$\displaystyle = \left(\frac{\lim_{x\to 1} 1}{\left(\lim_{x\to 1} 3\right)\left(\lim_{x\to 1} x\right) - \left(\lim_{x\to 1} 1\right)}\right)^3$ (product law)

$\displaystyle = \left(\frac{1}{3\cdot 1 - 1}\right)^3$ basic laws

$\displaystyle = \left(\frac{1}{2}\right)^3$ arithmetic

$\displaystyle = \frac{1}{8}$ arithmetic

② a) $\lim_{x\to3}\left(f(x)^2+3g(x)\right)$

$= \left(\lim_{x\to3} f(x)^2\right) + \left(\lim_{x\to3} 3g(x)\right)$ sum law

$= \left(\lim_{x\to3} f(x)\right)^2 + \left(\lim_{x\to3} 3\right)\left(\lim_{x\to3} g(x)\right)$ power law and product law

$= 5^2 + 3\cdot(-2)$ values given for $\lim_{x\to3} f(x)$ and $\lim_{x\to3} g(x)$

$= 19$ arithmetic

b) $\lim_{x\to3} \dfrac{h(x)}{f(x)+g(x)} = \dfrac{\lim_{x\to3} h(x)}{\left(\lim_{x\to3} f(x)+g(x)\right)}$ quotient law

$= \dfrac{\lim_{x\to3} h(x)}{\left(\lim_{x\to3} f(x)\right)+\left(\lim_{x\to3} g(x)\right)}$ sum law

$= \dfrac{4}{5+(-2)}$ values given

$= \dfrac{4}{3}$ arithmetic

c) $\lim_{x\to3}\left(x f(x) + x^2 h(x)\right) = \left(\lim_{x\to3} x f(x)\right) + \left(\lim_{x\to3} x^2 h(x)\right)$ sum law

$= \left(\lim_{x\to3} x\right)\left(\lim_{x\to3} f(x)\right) + \left(\lim_{x\to3} x^2\right)\left(\lim_{x\to3} h(x)\right)$ product law

$= \left(\lim_{x\to3} x\right)\left(\lim_{x\to3} f(x)\right) + \left(\lim_{x\to3} x\right)^2\left(\lim_{x\to3} h(x)\right)$ power law

$= 3\cdot5 + 3^2\cdot4$ basic laws and given values

$= 15 + 36 = \underline{\underline{51}}$ arithmetic

(3) a) $\lim_{x\to 0}\big(f(x)+g(x)\big) = +\infty$. As $x \to 0$, The value of $f(x)$ gets larger and larger [in a positive direction]. Adding a number close To L To a big positive number is sTill a big positive number. So $f(x)+g(x) \longrightarrow +\infty$.

b) $\lim_{x\to 0}\big(f(x)+g(x)\big) = +\infty$, because adding Two big positive numbers gives an even bigger positive number.

c) $\lim_{x\to 0}\left(\frac{1}{x^2}+\frac{-2}{x^2}\right) = \lim_{x\to 0}\frac{-1}{x^2} = -\infty$

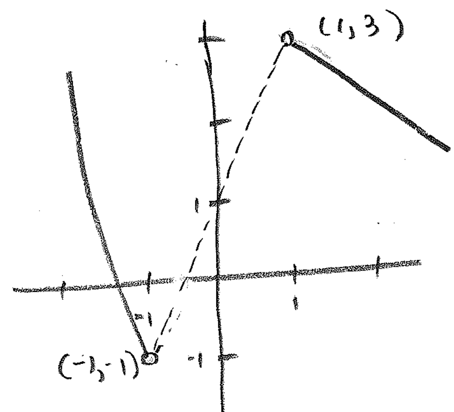
$\lim_{x\to 0}\left(\frac{2}{x^2}+\frac{-1}{x^2}\right) = \lim_{x\to 0}\frac{1}{x^2} = \infty$

$\lim_{x\to \infty}\left(\left(17+\frac{1}{x^2}\right)-\left(\frac{1}{x^2}\right)\right) = \lim_{x\to 0} 17 = 17$

(and we could replace 17 with any other number)

All of These limits are of The form "$(+\infty)+(-\infty)$", and what we see is That $\lim_{x\to 0} f(x)+g(x)$ could be literally anything. So There is no simple rule That applies To limits of The form $+\infty+(-\infty)$.

(4) a)



$\lim_{x\to -1^-} f(x) = \lim_{x\to -1^-} x^2-2 = -1$

$\lim_{x\to -1^+} f(x) = \lim_{x\to -1^+} ax+b = -a+b$

so we need $-a+b=-1$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} 4-x = 3$

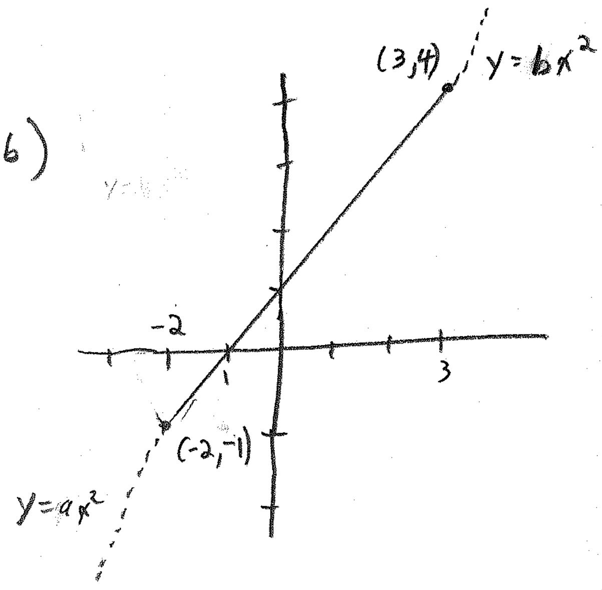
$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} ax+b = a+b$

so we need $a+b=3$

Solving $-a+b=-1$, $a+b=3$, adding we get $2b=2$ and $\underline{b=1}$. Then $a+1=3$ so $\underline{a=2}$.

Alternative approach: $y=ax+b$ is a line That must pass Through $(-1,-1)$ and $(1,3)$, so slope is $\underline{a=2}$ and intercept is $\underline{b=1}$

6)



$$\lim_{x\to -2^+} f(x) = \lim_{x\to -2^+} x+1 = -1$$

$$\lim_{x\to -2^-} f(x) = \lim_{x\to -2^-} ax^2 = 4a$$

So we need $4a = -1$ or $\underline{a = -\frac{1}{4}}$

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} x+1 = 4$$

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} bx^2 = 9b$$

So we need $9b = 4$ or $\underline{b = \frac{4}{9}}$

5) a) As x gets bigger and bigger, $\frac{1}{x}$ gets smaller and smaller, so $\lim_{x\to +\infty} \frac{1}{x} = 0$.

b) $\lim_{x\to +\infty} \left(\frac{x+1}{x}\right) = \lim_{x\to +\infty} \left(1+\frac{1}{x}\right) = 1$, because $\frac{1}{x} \to 0$.
(Or, when x is very large $x \approx x+1$, so $\frac{x+1}{x}$ is close to 1.)

c)

x	10	100	1000	10000	100000
$\sqrt{x^2+x} - x$	.48808	.49876	.499875	.4999875	.49999875

So it looks like $\lim_{x\to \infty} \left(\sqrt{x^2+x} - x\right) = \frac{1}{2}$

6) The two suggested estimates give the values 1.6 and 6.4. One idea is simply to take the average, giving $\underline{4}$ as the estimate. Another idea is to draw a careful graph and draw a tangent line at (1.5, 5.6). The slope of that line represents the velocity at time 1.5. When I did this, my slope was about $\underline{3.2}$.

7) If the object moves at constant velocity 1.2 for time 0.3, it travels a distance of $1.2 \times 0.3 = 0.36$, giving position $8.7 + 0.36$ or 9.06. Lacking more information, this is the best estimate.