This lab is to be done in groups of three students. You are not required to use the same groups as last week. Lab reports for this lab are due in class this Friday. Remember to show your work!

There will be a quiz at the start of class tomorrow.

1. Evaluate the following limits using algebra and the limit laws from Section 2.3. Do not simply "plug in." Apply one law at a time, or one algebraic manipulation.

a) $\lim\limits_{x\to 2} \dfrac{x^2 - x - 2}{x^2 + x - 6}$ **b)** $\lim\limits_{x\to -1} \dfrac{x^2 + 2x + 1}{x^2 + x}$ **c)** $\lim\limits_{x\to 1} \left(\dfrac{x-1}{3x^2 - 4x + 1}\right)^3$

2. Suppose that $\lim\limits_{x\to 3} f(x) = 5$, $\lim\limits_{x\to 3} g(x) = -2$, and $\lim\limits_{x\to 3} h(x) = 4$. Evaluate the following limits by applying laws. Show the full sequence of steps where you apply the laws.

a) $\lim\limits_{x\to 3} \left(f(x)^2 + 3g(x)\right)$ **b)** $\lim\limits_{x\to 3} \dfrac{h(x)}{f(x) + g(x)}$ **c)** $\lim\limits_{x\to 3} \left(xf(x) + x^2 h(x)\right)$

3. When applying a limit law such as $\lim\limits_{x\to 0}(f(x) + g(x)) = (\lim\limits_{x\to 0} f(x)) + (\lim\limits_{x\to 0} g(x))$, it is important to remember the assumption that the limits on the right exist. This means that the rules that we have seen do not apply to limits that don't exist because they are infinite. However, some things are still true in the infinite case.

a) Suppose that $\lim\limits_{x\to 0} f(x) = +\infty$ and $\lim\limits_{x\to 0} g(x) = L$, where L is some real number. What can you say in this case about $\lim\limits_{x\to 0}(f(x) + g(x))$? Explain your reasoning!

b) Suppose that $\lim\limits_{x\to 0} f(x) = +\infty$ and $\lim\limits_{x\to 0} g(x) = +\infty$. What can you say in this case about $\lim\limits_{x\to 0}(f(x) + g(x))$? Explain your reasoning!

c) Now suppose that $\lim\limits_{x\to 0} f(x) = +\infty$ and $\lim\limits_{x\to 0} g(x) = -\infty$. Each of the following pairs of functions have this property: $f(x) = \frac{1}{x^2}$, $g(x) = -\frac{2}{x^2}$; $f(x) = \frac{2}{x^2}$, $g(x) = -\frac{1}{x^2}$; $f(x) = 17 + \frac{1}{x^2}$, $g(x) = -\frac{1}{x^2}$. For each pair of functions, find $\lim\limits_{x\to 0}(f(x) + g(x))$. What do you learn by considering these examples?

4. For each problem, find values for the constants a and b, if possible, so that the limits $\lim\limits_{x\to t} f(x)$ exists for every number t. Draw a graph of the function. Explain your reasoning!

a) $f(x) = \begin{cases} x^2 - 2, & \text{if } x < -1 \\ ax + b, & \text{if } -1 \le x \le 1 \\ 4 - x, & \text{if } x > 1 \end{cases}$ **b)** $f(x) = \begin{cases} ax^2, & \text{if } x \le -2 \\ x + 1, & \text{if } -2 < x < 3 \\ bx^2, & \text{if } x \ge 3 \end{cases}$

5. We have been looking at limits of the form $\lim\limits_{x\to a} f(x)$. It is also possible to define limits "at infinity," $\lim\limits_{x\to +\infty} f(x)$. This type of limit asks, what happens to the value of $f(x)$ "in the limit" as the value of x gets larger and larger.

a) Explain why $\lim\limits_{x\to +\infty} \frac{1}{x} = 0$.

b) What is $\lim_{x\to+\infty} \frac{x+1}{x}$? Why?

c) Consider the function $f(x) = \sqrt{x^2+x} - x$, For this function, make a table showing the value of the function for $x = 1$, $x = 10$, $x = 100$, $x = 1000$, $x = 10000$, and $x = 100000$. Based on the table, what do you expect $\lim_{x\to+\infty} (\sqrt{x^2+x} - x)$ to be?

6. We have discussed the problem of "instantaneous velocity" at a given time, and we have seen that average velocity over an interval can be used to approximate instantaneous velocity. Suppose that the position of some object at time t is given by a function $f(t)$. The values of $f(t)$ are measured for certain values of t, as shown in the following table:

t	1.0	1.25	1.5	1.75	2.0
$f(t)$	1.5	4.0	5.6	6.0	5.0

Given just the data in this table, you can't be sure what the instantaneous velocity at time $t = 1.5$ is. But you can still give an estimate. Possible estimates could be $\frac{f(1.75)-f(1.5)}{1.75-1.5}$ or $\frac{f(1.5)-f(1.25}{1.5-1.25}$, but these are not necessarily the **best** estimates that can be given based on the available data. Why not? How could you get a better estimate? Ideally, you should come up with and compare a couple of different approaches to this problem. What is your best estimate for the instantaneous velocity of the object at $t = 1.5$? Explain! (There is no perfect solution to this problem.)

7. Suppose that you manage to measure the position and velocity of an object at time $t = 3.7$. The position is 8.7, and the velocity is 1.2. Estimate the position of the object at time $t = 4$. Explain why your estimate is only an approximation. Is there any particular kind of motion for which your estimate would be exactly correct?