Math 130-01, Spring 2020 Sample Answers for Lab #3

① a) $p(\frac{1}{2}) = 3\cdot(\frac{1}{2})^5 - \frac{1}{2} - 1 = \frac{3}{32} - \frac{16}{32} - \frac{32}{32} = -\frac{45}{32}$. Since $p(\frac{1}{2}) < 0$ and $p(1) > 0$, The IVT says There is a root in The interval $[\frac{1}{2}, 1]$.

b) To narrow down to an interval of length $\frac{1}{4}$, look at $p(\frac{3}{4})$. It will be either positive or negative. If positive, Then by The IVT, The root will be in $[\frac{1}{2}, \frac{3}{4}]$, because $f(\frac{1}{2})$ is negative. If $p(\frac{3}{4}) < 0$, The root is in $[\frac{3}{4}, 1]$ because $p(1) > 0$. (If $p(\frac{3}{4})$ happens to be zero exactly, Then we know That The root is 0 exactly.) In fact, by calculator, $p(\frac{3}{4}) = -1.038\ldots$. Since $p(\frac{3}{4}) < 0$ and $p(1) > 0$, The root is in $[\frac{3}{4}, 1]$

c) Look at $x = \frac{7}{8}$, which is half way between $\frac{3}{4}$ and 1. $p(\frac{7}{8}) = -0.336\ldots$. Since $p(\frac{7}{8}) < 0$ and $p(1) > 0$, The root is in The interval $[\frac{7}{8}, 1]$. [On The next step, we would find it is in The range $[\frac{7}{8}, \frac{15}{16}]$, since $p(\frac{15}{16}) > 0$ and $p(\frac{7}{8}) < 0$.]

d) Start with $a = 0$, $b = 1$. On each step, choose The number c That is halfway between a and b. ($c = \frac{a+b}{2}$.) If $p(c)$ has The same sign as $p(a)$, Then The root is in $[c, b]$, so replace a with c; if $p(c)$ has The same sign as $p(a)$, Then The root is is in $[a, c]$, so replace b with c. If The new $[a, b]$ has a length That is smaller Than The desired accuracy, stop. Otherwise continue on To The next step.

This works because at every step, The root is known To be in $[a, b]$, and when we stop we can Take $\frac{a+b}{2}$ to be The approximation That we want.

(2) a) Let D be the distance from the car to the lake. Then $f(0) = 0$, $f(2) = D$, $g(0) = D$, $g(2) = 0$.

b) $h(0) = f(0) - g(0) = 0 - D = -D$

$h(2) = f(2) - g(2) = D - 0 = D$

c) By the IVT, because 0 is between $-D$ and D, there must be a c in $[0, 2]$ such that $h(c) = 0$. Then $f(c) = g(c)$, which means you are the same distance from the car at c hours after 7:00 Friday as at c hours after 7:00 on Sunday.

(3) $f(x) = \dfrac{|x^2-9|}{x^2+x-12} = \dfrac{|x^2-9|}{(x+4)(x-3)}$. The only points of discontinuity are $x = 3$ and $x = -4$, where the denominator is 0. At $x = -4$, $\dfrac{|x^2-9|}{(x+4)(x-3)}$ has the form $\frac{5}{0}$, so the limit of $f(x)$ from the left or right as $x \to -4$ is infinite. $f(x)$ has an infinite discontinuity at $x = 4$. At $x = 3$, we need to examine the limits from the left and from the right. Note that $f(x) = \dfrac{|(x-3)(x+3)|}{(x+4)(x-3)}$

$= \dfrac{|x+3|}{x+4} \cdot \dfrac{|x-3|}{x-3}$. $\lim\limits_{x \to 3^-} f(x) = \lim\limits_{x \to 3^-} \dfrac{|x+3|}{x+4} \cdot \dfrac{-(x-3)}{x-3} = \frac{6}{7} \cdot (-1) = -\frac{6}{7}$

because for $x < 3$, $x - 3 < 0$ and $|x-3| = -(x-3)$. And

$\lim\limits_{x \to 3^+} f(x) = \lim\limits_{x \to 3^+} \dfrac{|x+3|}{|x+4|} \cdot \dfrac{x-3}{x-3} \cdot \frac{6}{7} \cdot 1 = \frac{6}{7}$, since for $x > 3$, $x - 3 > 0$ and $|x-3| = x - 3$. Since limits from left and right are different, it's a jump discontinuity.

4. a) Let $\varepsilon > 0$. Let $\delta = \frac{\varepsilon}{5}$. We then have that if $0 < |x-5| < \delta$, then $|f(x) - L| = |(5x-10) - 15| = |5x - 25|$

$$= 5|x-5| < 5 \cdot \frac{\varepsilon}{5} < \varepsilon.$$

b) Let $\varepsilon > 0$. Let $\delta = \frac{\varepsilon}{3}$. We then have that if $0 < |x-2| < \delta$, then $|f(x) - L| = |(-3x+1) - (-5)|$

$$= |-3x+6| = |(-3)(x-2)| = |3| \cdot |x-2| < 3 \cdot \frac{\varepsilon}{3} = \varepsilon.$$

5) a) $$\lim_{x \to 2} \frac{\sqrt{x} - \sqrt{2}}{x^2 - 2x} = \lim_{x \to 2} \frac{\sqrt{x} - \sqrt{2}}{x^2 - 2x} \cdot \frac{\sqrt{x} + \sqrt{2}}{\sqrt{x} + \sqrt{2}}$$

$$= \lim_{x \to 2} \frac{x-2}{x(x-2)(\sqrt{x} + \sqrt{2})} = \lim_{x \to 2} \frac{1}{x(\sqrt{x} + \sqrt{2})} = \frac{1}{2(\sqrt{2} + \sqrt{2})} = \frac{1}{4\sqrt{2}}$$

b) $$\lim_{x \to 7} \frac{\sqrt{x+2} - 3}{\sqrt{x-3} - 2} = \lim_{x \to 7} \frac{\sqrt{x+2} - 3}{\sqrt{x-3} - 2} \cdot \frac{\sqrt{x+2} + 3}{\sqrt{x+2} + 3} \cdot \frac{\sqrt{x-3} + 2}{\sqrt{x-3} + 2}$$

$$= \lim_{x \to 7} \frac{[(\sqrt{x+2} - 3)(\sqrt{x+2} + 3)](\sqrt{x-3} + 2)}{[(\sqrt{x-3} - 2)(\sqrt{x-3} + 2)](\sqrt{x+2} - 3)} = \lim_{x \to 7} \frac{(x+2-3^2)(\sqrt{x-3} + 2)}{(x-3-2^2)(\sqrt{x+2} - 3)}$$

$$= \lim_{x \to 7} \frac{(x-7)(\sqrt{x-3} + 2)}{(x-7)(\sqrt{x+2} + 3)} = \frac{\sqrt{7-3} + 2}{\sqrt{7+2} + 3} = \frac{\sqrt{4} + 2}{\sqrt{9} + 3} = \frac{2+2}{3+3} = \frac{2}{3}$$

c) $$\lim_{x \to 1} \left(\frac{2}{(x-3)(x-1)} - \frac{1}{(x-2)(x-1)} \right)$$

$$= \lim_{x \to 1} \left(\frac{2(x-2) - 1 \cdot (x-3)}{(x-3)(x-1)(x-2)} \right) = \lim_{x \to 1} \left(\frac{2x - 4 - x + 3}{(x-3)(x-1)(x-2)} \right)$$

$$= \lim_{x \to 1} \left(\frac{x-1}{(x-3)(x-1)(x-2)} \right) = \lim_{x \to 1} \frac{1}{(x-3)(x-2)}$$

$$= \frac{1}{(1-3)(1-2)} = \frac{1}{(-2)(-1)} = \frac{1}{2}$$