This lab is to be done in groups of three students. Lab reports for this lab are due in class this Friday. Remember to show your work! ***Expect another quiz in class tomorrow—and a test one week later!***

1. In class, we used the Intermediate Value Theorem to show that the polynomial $p(x) = 3x^5 - x - 1$ has at least one root somewhere in the interval $[0, 1]$, because $p(0) < 0$ and $p(1) > 0$. In fact, it has only one root in that interval. You can use a calculator for this exercise.

a) Find the value of $p\left(\frac{1}{2}\right)$. Is the root of $p(x)$ in the interval $\left[0, \frac{1}{2}\right]$, or is it in the interval $\left[\frac{1}{2}, 1\right]$? Why?

b) Find an interval of length $\frac{1}{4}$ that contains the root. Justify your answer.

c) Find intervals of length $\frac{1}{8}$ and of length $\frac{1}{16}$ that contains the root.

d) Explain carefully how you can use the Intermediate Value Theorem to find an approximation for the root that is accurate to any desired degree of accuracy. Specify the procedure for finding the approximate value of the root: What do you do at each step? When do you stop? Explain why the procedure works.

2. [From another calculus textbook.] Suppose that you park your car at the start of trail at a national park 7:00 AM on a Friday morning, and you make a two-hour hike to a lake, arriving at 9:00 AM. At 7:00 AM on Sunday morning, you leave the lake and make the two-hour hike back to your car, arriving at 9:00 AM.

Let $f(t)$ be your distance from the car t hours after 7:00 AM on Friday, and let $g(t)$ be your distance from the car t hours after 7:00 AM on Sunday.

a) Find values for $f(0)$, $f(2)$, $g(0)$, $g(2)$. (Your answers will involve an unknown constant.)

b) Let $h(t) = f(t) - g(t)$. Find $h(0)$ and $h(2)$.

c) Use the Intermediate Value Theorem to conclude that there must be some point along the trail that you will pass at exactly the same time on Friday and on Sunday.

3. Let $f(x) = \dfrac{|x^2 - 9|}{x^2 + x - 12}$. Find all values of x where the function is **not** continuous, and classify each such value as a removable discontinuity, a jump discontinuity, or an infinite discontinuity. Justify your answers!

4. We have just covered the epsilon-delta definition of limit: $\lim\limits_{x\to a} f(x) = L$ if $f(x)$ is defined on some open interval that contains a, except possibly at a itself, and for every $\epsilon > 0$, there is a $\delta > 0$ such that if $0 < |x - a| < \delta$ then $|f(x) - L| < \epsilon$.

a) Use this definition to prove directly that $\lim\limits_{x\to 5}(5x - 10) = 15$. You can follow the same outline as the proof of similar examples given in class and in the textbook.

b) Use the definition to prove directly that $\lim\limits_{x\to 2}(-3x + 1) = -5$.

5. Evaluate the following limits by first applying any necessary algebraic techniques, such as adding fractions or using the method of "rationalizing" the numerator and/or denominator. (For part c, factor the denominators first!)

a) $\lim\limits_{x\to 2} \dfrac{\sqrt{x} - \sqrt{2}}{x^2 - 2x}$ **b)** $\lim\limits_{x\to 7} \dfrac{\sqrt{x+2} - 3}{\sqrt{x-3} - 2}$ **c)** $\lim\limits_{x\to 1} \left(\dfrac{2}{x^2 - 4x + 3} - \dfrac{1}{x^2 - 3x + 2}\right)$