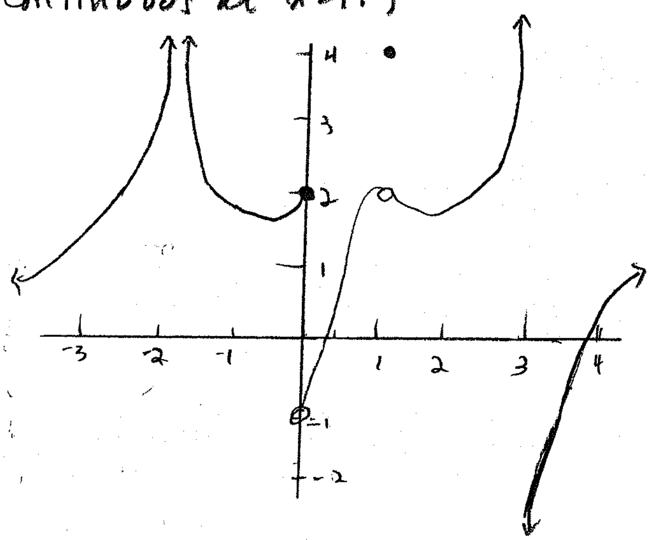
Math 130-01, Spring 2020 Sample Answers To Lab #4

① As $x \to +\infty$ it looks like the function values are approaching 7, so the guess is $\lim_{x\to+\infty} f(x) = 7$. Similarly, $\lim_{x\to-\infty} f(x) = 9$.

$\lim_{x\to -1^-} f(x) = 3$, but there is no information about $\lim_{x\to -1^+} f(x)$.

$\lim_{x\to 1^-} f(x) = 2.25 = \lim_{x\to 1^+} f(x)$, so $\lim_{x\to 1} f(x) = 2.25 = f(1)$. (So, $f(x)$ is continuous at $x=1$.) And it looks like $\lim_{x\to 2^+} f(x) = +\infty$.

②



③ $\lim_{x\to -2^-} f(x) = \lim_{x\to -2^-} 1-x^2 = -3$

$\lim_{x\to -2^+} f(x) = \lim_{x\to -2^+} 2x+1 = -3$

So $\lim_{x\to -2} f(x) = -3$. Also, $f(-2) = -3$

so $f(x)$ is continuous at -2.

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} 2x+1 = 5$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^-} 3 = 3$

So $\lim_{x\to 2} f(x)$ DNE; not continuous

$\lim_{x\to -2^-} g(x) = \lim_{x\to -2^-} -4(x+1) = 4$

$\lim_{x\to -2^+} g(x) = \lim_{x\to -2^+} x^2 = 4$

$\lim_{x\to 2} g(x) = 4 = g(2)$; continuous

$$\left.\begin{aligned} \lim_{x\to 2^-} g(x) &= \lim_{x\to 2^-} x^2 = 4 \\ \lim_{x\to 2^+} g(x) &= \lim_{x\to 2^+} 2x = 4 \end{aligned}\right\}$$ $\lim_{x\to 2} g(x) = 4$ but $g(4) \neq 4$; not continuous

④ A function cannot be differentiable where it is not continuous, so neither $f'(2)$ nor $g'(2)$ exist. To compute $f'(-2)$, $g'(-2)$, we need to compute left and right limits separately.

$$\lim_{x\to -2^-} \frac{f(x)-f(-2)}{x-(-2)} = \lim_{x\to -2^-} \frac{(1-x^2)-(-3)}{x+2}$$

$= \dots = 4$; $\lim_{x\to -2^+} \frac{f(x)-f(-2)}{x-(-2)}$

$= \lim_{x\to -2^+} \frac{(2x+1)-(-3)}{x+2} = \dots = 2$.

Limits from left and right are different, so $\lim_{x\to -2} \frac{f(x)-f(-2)}{x-(-2)}$ does not exist. That is, $\underline{f'(-2) \text{ DNE}}$.

For $g(x)$, $\lim_{x\to -2^-} \frac{g(x)-g(-2)}{x-(-2)}$

$= \lim_{x\to -2^-} \frac{-4(x+1)-4}{x+2} = \dots = -4$,

$\lim_{x\to -2^+} \frac{g(x)-g(-2)}{x-(-2)} = \lim_{x\to -2^+} \frac{x^2-4}{x+2} = -4$

so $\lim_{x\to -2} \frac{g(x)-g(-2)}{x-(-2)} = \underline{-4 = f'(-2)}$,

⑤ a) $\lim_{x\to 3} x^2+1 = 3^2+1 = 10$

b) $\lim_{t\to 1} \frac{t^3-t^2}{(t-1)^2} = \lim_{t\to 1} \frac{t^2(t-1)}{(t-1)^2}$

$= \lim_{t\to 1} \frac{t^2}{t-1}$. This is of the form $\frac{1}{0}$, so the limit $\underline{\text{DNE}}$.

5) c) $\lim_{x\to4}\frac{\sqrt{2x+1}-3}{x-4}=\lim_{x\to4}\frac{(\sqrt{2x+1}-3)(\sqrt{2x+1}+3)}{(x-4)(\sqrt{2x+1}+3)}=\lim_{x\to4}\frac{(2x+1)-9}{(x-4)(\sqrt{2x+1}+3)}$

$=\lim_{x\to4}\frac{2x-8}{(x-4)(\sqrt{2x+1}+3)}=\lim_{x\to4}\frac{2(x-4)}{(x-4)(\sqrt{2x+1}+3}=\frac{2}{\sqrt{2\cdot4+1}+3}=\frac{2}{6}=\frac{1}{3}$

d) $\lim_{x\to2}\frac{x^2-7x+10}{2x^2+x-10}=\lim_{x\to2}\frac{(x-2)(x-5)}{(x-2)(2x+5)}=\lim_{x\to2}\frac{x-5}{2x+5}=\frac{-3}{9}=-\frac{1}{3}$

6) a) $f'(1)=\lim_{x\to1}\frac{f(x)-f(1)}{x-1}=\lim_{x\to1}\frac{(x+1)^2-4}{x-1}=\lim_{x\to1}\frac{x^2+2x+1-4}{x-1}$

$=\lim_{x\to1}\frac{x^2+2x-3}{x-1}=\lim_{x\to1}\frac{(x-1)(x+3)}{x-1}=\lim_{x\to1}(x+3)=4$

b) $g'(0)=\lim_{z\to0}\frac{(3z+2)-2}{z-0}=\lim_{z\to0}\frac{3z}{z}=\underline{\underline{3}}$ $\left[\text{OR }\lim_{x\to0}\frac{(3x+2)-2}{x-0},\text{ OR }\lim_{h\to0}\frac{3(0+h)+2-2}{h}\right]$

c) $S'(4)=\lim_{t\to4}\frac{\frac{1}{t-2}-\frac{1}{2}}{t-4}=\lim_{t\to4}\frac{1}{t-4}\left(\frac{2-(t-2)}{2(t-2)}\right)=\lim_{t\to4}\frac{1}{t-4}\left(\frac{4-t}{2(t-2)}\right)$

$=\lim_{t\to4}\frac{1}{t-4}\left(\frac{-(t-4)}{2(t-2)}\right)=\lim_{t\to4}\frac{-1}{2(t-2)}=\frac{-1}{4}$

d) $t'(3)=\lim_{x\to3}\frac{t(x)-t(3)}{x-3}=\lim_{x\to3}\frac{\sqrt{7x+4}-5}{x-3}\cdot\frac{\sqrt{7x+4}+5}{\sqrt{7x+4}+5}$

$=\lim_{x\to3}\frac{7x+4-25}{(x+3)(\sqrt{7x+4}+5)}=\lim_{x\to3}\frac{7x-21}{(x+3)(\sqrt{7x+4}+5)}=\lim_{x\to3}\frac{7}{\sqrt{7x+4}+5}=\frac{7}{10}$

7) Average velocity $=\frac{S(2)-S(0)}{2-0}=\frac{(2\cdot2^2+3\cdot2)-0}{2}=\frac{14}{2}=\underline{\underline{7}}$

velocity at $t=0$ is $S'(0)=\lim_{t\to0}\frac{S(t)-S(0)}{t-0}=\lim_{t\to0}\frac{2t^2+3t-0}{t}=\lim_{t\to0}2t+3=\underline{\underline{3}}$

8) Yes, of course. what happens to the left of a need not have anything to do with what happens to the right. Eg; $f(x)=\begin{cases}1/x, & x>0\\ 3x, & x\le0\end{cases}$ at $x=0$.

9) when finding a tangent line to $y=f(x)$ at the point $(a,f(a))$, we only know one point on the line, namely $(a,f(a))$. To find the slope of a line, we need two points. The solution is to look at a secant line through $(a,f(a))$ and $(x,f(x))$, where x is close to a. The slope, $\frac{f(x)-f(a)}{x-a}$ is an approximation for the slope of the tangent line. The exact slope is gotten by taking a limit: $\lim_{x\to a}\frac{f(x)-f(a)}{x-a}=f'(a)$.