This lab is meant as review for tomorrow's test. It will not be collected. Sample answers will be available at the lab. Note that this is meant to be a general review of the material you should know; however, not every type of problem from this lab will appear on the test and not every problem on the test will be similar to a problem on this lab. **Also, remember that you should be able to state important definitions and theorems!**

1. Consider the following table that shows some values of a function $f(x)$. Make plausible guesses for whatever limits you can, based on the information in the table, including one-sided limits and limits at infinity. (Limits at infinity were covered on a previous lab but will not be on the test.)

x	$f(x)$
-1000	9.002
-100	9.15
-10	9.2
-1.1	3.207
-1.01	3.065
-1.001	3.013
-1.0001	3.002
-1	2

x	$f(x)$
0.75	2.470
0.9	2.284
0.99	2.2512
0.999	2.2501
1	2.25
1.001	2.2495
1.01	2.238
1.1	2.225

x	$f(x)$
2.1	1.7
2.01	16.25
2.001	1581
10	7
100	7.1
1000	6.94
10000	7.002
100000	6.999

2. Draw a graph of a function $y = h(x)$ that satisfies all of the following:

a) For all values of a not mentioned, $h(a)$ and $\lim_{x\to a} h(x)$ are both defined and are equal

b) $h(1) = 4$ **c)** $\lim_{x\to 1^-} h(x) = 2$ **d)** $\lim_{x\to 1^+} h(x) = 2$

e) $\lim_{x\to 3^-} h(x) = +\infty$ **f)** $\lim_{x\to 3^+} h(x) = -\infty$ **g)** $\lim_{x\to -2} h(x) = +\infty$

h) $h(0) = 2$ **i)** $\lim_{x\to 0^-} h(x) = 2$ **j)** $\lim_{x\to 0^+} h(x) = -1$

3. For each of the following functions, investigate the behavior of limits of the function at $x = -2$ and $x = 2$. Also determine whether the function is continuous at $x = -2$ and at $x = 2$.

$$f(x) = \begin{cases} 1 - x^2, & \text{if } x < -2 \\ 2x + 1, & \text{if } -2 \le x \le 2 \\ 3, & \text{if } x > 2 \end{cases} \qquad g(x) = \begin{cases} -4(x+1), & \text{if } \le -2 \\ x^2, & \text{if } -2 < x < 2 \\ 3, & \text{if } x = 2 \\ 2x, & \text{if } x > 2 \end{cases}$$

4. For each of the functions in the previous problem, determine whether the function is differentiable at -2 and whether it is differentiable at 2. If so, find the value of the derivative.

5. Evaluate the following limits. (This means show any algebraic work, but you do not have to show the application of individual limit laws.)

a) $\lim_{x\to 3} x^2 + 1$ **b)** $\lim_{t\to 1} \dfrac{t^3 - t^2}{(t-1)^2}$ **c)** $\lim_{x\to 4} \dfrac{\sqrt{2x+1} - 3}{x - 4}$ **d)** $\lim_{x\to 2} \dfrac{x^2 - 7x + 10}{2x^2 + x - 10}$

6. Use the definition of derivative to compute the following derivatives directly:

a) $f'(1)$ where $f(x) = (x+2)^2$

b) $g'(0)$ where $g(z) = 3z + 2$

c) $s'(4)$ where $s(t) = \frac{1}{t-2}$

d) $t'(3)$ where $t(x) = \sqrt{7x+4}$

7. Suppose that the position of a point at time t is given by $s(t) = 2t^2 + 3t$. What is its average velocity between time $t = 0$ and time $t = 2$? What is its instantaneous velocity at $t = 0$?

8. Is it possible for $\lim_{x \to a^-} f(x)$ to exist while $\lim_{x \to a^+} f(x)$ does not exist? (Explain!)

9. Write an essay discussing the relationship between derivatives and tangent lines.