Math 130-01, Spring 2020 Sample Answers to Lab #5

① a) $\frac{d}{dx}(\sin(3x^2)+x) = \left(\frac{d}{dx}\sin(3x^2)\right) + \left(\frac{d}{dx}x\right)$ sum rule

b) $\frac{d}{dx}\left(3\sin(3x^2+x)\right) = 3\cdot\frac{d}{dx}\sin(3x^2+x)$ constant multiple rule

c) $\frac{d}{dx}(\sin(3x^2+x))$ none of the available rules apply

d) $\frac{d}{dx}\left(\frac{\sin(x)}{\cos(x)}\right) = \frac{\cos(x)\frac{d}{dx}\sin(x) - \sin(x)\frac{d}{dx}\cos(x)}{(\cos(x))^2}$ quotient rule

e) $\frac{d}{dx}\left(\frac{\sqrt[3]{x^3-3x+1}}{x^2}\right) = \frac{x^2\frac{d}{dx}\sqrt[3]{x^3-3x+1} - \sqrt[3]{x^3-3x+1}\,\frac{d}{dx}(x^2)}{(x^2)^2}$ quotient rule

f) $\frac{d}{dx}(\sec(x)\tan(x)) = \sec(x)\frac{d}{dx}\tan(x) + \tan(x)\frac{d}{dx}(\sec(x))$ product rule

g) $\frac{d}{dx}(x^2e^{x^2+1}) = x^2\frac{d}{dx}e^{x^2+1} + e^{x^2+1}\frac{d}{dx}(x^2)$ product rule

h) $\frac{d}{dx}\left(\frac{1}{x}-\frac{1}{x^2}\right) = \frac{d}{dx}\left(\frac{1}{x}\right) - \frac{d}{dx}\left(\frac{1}{x^2}\right)$ difference rule

② $f'(0) = \lim_{x\to 0}\frac{f(x)-f(0)}{x-0} = \lim_{x\to 0}\frac{x|x|-0}{x-0} = \lim_{x\to 0}|x| = 0.$

[The last limit was done in class by looking at $\lim_{x\to 0^+}|x|$ and $\lim_{x\to 0^-}|x|$.] To show $f'(x) = x|x|$ for all x, look at the three cases $x<0$, $x=0$, $x>0$.

For $x<0$, $|x| = -x$ and $f(x) = x\cdot|x| = x\cdot(-x) = -x^2$, so $f'(x) = -2x = 2\cdot(-x) = 2\cdot|x|$.

For $x=0$, we have already shown $f'(0) = 0 = 2\cdot|0|$

For $x>0$, $|x| = x$, and $f(x) = x\cdot|x| = x\cdot x = x^2$, so $f'(x) = 2x = 2\cdot|x|$. So, the formula $f'(x) = 2|x|$ works in all cases.

③ a) $p(x) = 2x^3 - 3x^2 + \frac{1}{2}x - 1$

$p'(x) = 6x^2 - 6x + \frac{1}{2}$

$p''(x) = 12x - 6$

$p'''(x) = 12$

$p^{(4)}(x) = 0$

b) $q(x) = x^5 + 2x$

$q'(x) = 5x^4 + 2$

$q''(x) = 20x^3$

$q'''(x) = 60x^2$

$q^{(4)}(x) = 120x$

$q^{(5)}(x) = 120$

$q^{(6)}(x) = 0$

c) Every time you take the derivative of a polynomial, any constant terms disappear and all the exponents on the variables go down by one. Eventually, all the exponents will drop to zero, giving constant terms that will disappear on the next step.

④ a) $\frac{d}{dx}x^2 = \frac{d}{dx}x \cdot x = x \cdot \frac{d}{dx}x + x \cdot \frac{d}{dx}x = x \cdot 1 + x \cdot 1 = 2x$

b) $\frac{d}{dx}x^3 = \frac{d}{dx}x \cdot x^2 = x \cdot \frac{d}{dx}x^2 + x^2 \cdot \frac{d}{dx}x = x \cdot 2x + x^2 \cdot 1 = 2x^2 + x^2 = 3x^2$

c) $\frac{d}{dx}x^4 = \frac{d}{dx}x \cdot x^3 = x \cdot \frac{d}{dx}x^3 + x^3 \cdot \frac{d}{dx}x = x \cdot 3x^2 + x^3 \cdot 1 = 3x^3 + x^3 = 4x^3$

d) $\frac{d}{dx}x^k = \frac{d}{dx}x \cdot x^{k-1} = x \cdot \frac{d}{dx}x^{k-1} + x^{k-1}\frac{d}{dx}x = x \cdot (k-1)x^{k-2} + x^{k-1} \cdot 1$

$= (k-1)x^{k-1} + x^{k-1} = kx^{k-1}$

⑤ a+b) $\lim_{h \to 0} \frac{f(a+h) - f(a-h)}{2h} = \lim_{h \to 0} \frac{\sqrt{a+h} - \sqrt{a-h}}{2h} \cdot \frac{\sqrt{a+h} + \sqrt{a-h}}{\sqrt{a+h} + \sqrt{a-h}}$

$= \lim_{h \to 0} \frac{(a+h) - (a-h)}{2h(\sqrt{a+h} + \sqrt{a-h})} = \lim_{h \to 0} \frac{2h}{2h(\sqrt{a+h} + \sqrt{a-h})} = \frac{1}{\sqrt{a} + \sqrt{a}} = \frac{1}{2\sqrt{a}}$

c) $g'(0) = \lim_{h \to 0} \frac{g(0+h) - g(0-h)}{2h} = \lim_{h \to 0} \frac{|0+h| - |0-h|}{2h}$

$= \lim_{h \to 0} \frac{|h| - |-h|}{2h} = \lim_{h \to 0} \frac{0}{2h} = 0$, since $|h| = |-h|$.