This lab is due in class on Friday. Groups for this week's lab will be assigned randomly. Next week, you will be able to choose groups on your own again. There is no quiz this week, but there will be one next week.

1. Finding derivatives of formulas is a pretty mechanical step-by-step process, as long as you have memorized the rules (or have a table of formulas like the one two pages from the back of the textbook). The problem is to look at the formula and figure out which rule to apply. For that, you have to match the formula to one of the rules. Here is a short table of general rules, stated using the $\frac{d}{dx}$ notation:

Constant Multiple Rule:	$\frac{d}{dx}(c \cdot f(x)) = c \cdot \frac{d}{dx} f(x)$
Sum Rule:	$\frac{d}{dx}(f(x) + g(x)) = \frac{d}{dx}(f(x)) + \frac{d}{dx}(g(x))$
Difference Rule:	$\frac{d}{dx}(f(x) - g(x)) = \frac{d}{dx}(f(x)) - \frac{d}{dx}(g(x))$
Product Rule:	$\frac{d}{dx}(f(x) \cdot g(x)) = f(x) \cdot \frac{d}{dx}(g(x)) + g(x) \cdot \frac{d}{dx}(f(x))$
Quotient rule:	$\dfrac{d}{dx}\left(\dfrac{f(x)}{g(x)}\right) = \dfrac{g(x) \cdot \frac{d}{dx}(f(x)) - f(x) \cdot \frac{d}{dx}(g(x))}{(g(x))^2}$

For example, we can apply the quotient rule to show that

$$\frac{d}{dx}\left(\frac{3x^3 - \sqrt{x}}{\sin(x) + 7}\right) = \frac{(\sin(x) + 7) \cdot \frac{d}{dx}(3x^3 - \sqrt{x}) - (3x^3 - \sqrt{x}) \cdot \frac{d}{dx}(\sin(x) + 7)}{(\sin(x) + 7)^2}$$

For each of the following formulas, apply **one** of the above rules and show the exact result of applying that rule, **or** state that none of the rules apply to the formula. For each formula, you need to figure out the *last* operator that would be applied when evaluating the formula. That operator tells you which rule to apply. Note that you are only being asked to do **one step** of the process of computing the derivative; you are not being asked to work through to the final answer.

a) $\sin(3x^2) + x$ **b)** $3\sin(3x^2 + x)$ **c)** $\sin(3x^2 + x)$

d) $\dfrac{\sin(x)}{\cos(x)}$ **e)** $\dfrac{\sqrt[3]{x^3 - 3x + 1}}{x^2}$ **f)** $\sec(x)\tan(x)$

g) $x^2(e^{x^2+1})$ **h)** $x^2(e^{x^2+1}) + 1$ **i)** $\dfrac{1}{x} - \dfrac{1}{x^2}$

2. In class, we showed that the function $g(x) = |x|$ is not differentiable at $x = 0$, by looking at limits from the left and from the right. Now, let $f(x) = x|x|$. Show, using the definition of the derivative at 0, that $f'(0)$ exists and is equal to zero. You will have to compute the left and right limits separately, using the definition of $|x|$ as a split function. Then show that $f'(x) = 2|x|$ for all x by considering the cases $x < 0$, $x = 0$, and $x > 0$ separately, and again using the definition of $|x|$.

Lots more on the back!

3. We know that the derivative of a polynomial $p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ is given by the formula $p'(x) = a_n n x^{n-1} + a_{n-1}(n-1)x^{n-2} + \cdots + a_1$.

a) Let $p(x) = 2x^3 - 3x^2 + \frac{1}{2}x - 1$. Compute the first four derivatives of $p(x)$, That is compute $p'(x)$, $p''(x)$, $p'''(x)$, and $p^{(4)}(x)$.

b) Find the first six derivatives of $q(x) = x^5 + 2x$. That is, compute $q'(x)$ through $q^{(6)}(x)$.

c) If you take any polynomial and compute its first, second, third, ... derivatives, eventually you will end up with zero. Explain in words why this is true. How many derivatives do you have to take before you get zero? Why?

(Polynomials are the **only** functions for which this is true. That is, a function $f(x)$ is a polynomial if and only if there is some derivative $f^{(n)}(x)$ which is zero. But we won't be able to prove this until we study antiderivatives at the very end of the course.)

4. We know that $\frac{d}{dx}x^n = nx^{n-1}$, for $n = 2, 3, 4, \ldots$. This problem relates that formula to the product rule.

a) Consider the function x^2. Write x^2 as $x \cdot x$, and apply the product rule to $x \cdot x$ to verify that $\frac{d}{dx}x^2 = 2x$. (We already know that $\frac{d}{dx}x = 1$, and you can use that fact.)

b) Consider the function x^3. Write x^3 as $x \cdot x^2$, and apply the product rule to $x \cdot x^2$ to verify that $\frac{d}{dx}x^3 = 3x^2$. Use the formula for $\frac{d}{dx}x^2$ from part **a)**.

c) Consider x^4. Write x^4 as $x \cdot x^3$, and apply the product rule to $x \cdot x^3$ to verify that $\frac{d}{dx}x^4 = 4x^3$.

d) Now, suppose that k is some integer greater than 4 and that you already know the formula $\frac{d}{dx}x^{k-1} = (k-1)x^{k-2}$ for that particular number k. Consider x^k. Write x^k as $x \cdot x^{k-1}$, and apply the product rule to $x \cdot x^{k-1}$ to verify that $\frac{d}{dx}x^k = kx^{k-1}$. (Hopefully, this convinces you that $\frac{d}{dx}x^n = nx^{n-1}$ works in general.)

5. We have defined the derivative of a function $f(x)$ at $x = a$ to be the slope of the tangent line to the graph $y = f(x)$ at the point $(a, f(a))$. This slope can be computed as the limit of the slope of the secant line between the point $(a, f(a))$ and the point $(a+h, f(a+h))$, as $h \to 0$. Suppose that we decided, instead, to use two points on opposite sides of $(a, f(a))$ to make a secant line. That is, consider the secant line between the points $(a-h, f(a-h))$ and $(a+h, f(a+h))$. If you draw different types of secant lines on some graphs, you might think that a secant line through $(a-h, f(a-h))$ and $(a+h, f(a+h))$ gives a better estimate of the slope of the tangent line than a secant line that passes through $(a, f(a))$,

a) Consider a secant line through the points $(a-h, f(a-h))$ and $(a+h, f(a+h))$. What is the slope of this secant line, and what formula do you get for $f'(x)$ by taking the limit of that slope as $h \to 0$?

b) Use your formula to compute the derivative function of the function $f(x) = \sqrt{x}$. You should get the same answer that we get using the usual definition of the derivative, that is, $f'(x) = \frac{1}{2\sqrt{x}}$.

c) Suppose that you try to use the nifty new formula to compute the derivative of the function $g(x) = |x|$ at $x = 0$. Do it! The answer should be zero. Draw a picture (with an explanation in words) to illustrate why this happens. But as we have shown, $g'(0)$ does not exist for this function (using the normal definition)! So the new definition is not really equivalent to the old one. (It can be shown however that if the derivative **does** exist according to the regular definition, then the new definition gives the same answer.)