Math 130-01, Spring 2020 Sample Answers To Lab #6

1 a) $\frac{d}{dt}\left(\tan(t)+5\cos(t)\right) = \frac{d}{dt}\tan(t) + 5\frac{d}{dt}\cos(t) = \sec^2(t) - 5\sin(t)$

b) $\frac{d}{dx}\frac{1}{2+\sin(x)} = -\frac{1}{(2+\sin(x))^2}\cdot\frac{d}{dx}(2+\sin(x)) = -\frac{\cos(x)}{(2+\sin(x))^2}$

c) $\frac{d}{d\theta}\frac{1+\sec(\theta)}{1+\csc(\theta)} = \frac{(1+\csc(\theta))\frac{d}{d\theta}(1+\sec(\theta)) - (1+\sec(\theta))\frac{d}{d\theta}(1+\csc(\theta))}{(1+\csc(\theta))^2}$

$= \frac{(1+\csc(\theta))\cdot\sec\theta\tan\theta - (1+\sec\theta)\cdot(-\csc\theta\cot\theta)}{(1+\csc\theta)^2}$

d) $\frac{d}{dx}(x\sec x\tan x) = x\frac{d}{dx}\sec x\tan x + \sec x\tan x\cdot\frac{d}{dx}x$

$= x\left[\sec x\cdot\frac{d}{dx}\tan x + \tan x\frac{d}{dx}\sec x\right] + \sec x\tan x$

$= x\left[\sec x\cdot\sec^2 x + \tan x\cdot\sec x\tan x\right] + \sec x\tan x$

$= x\sec^3 x + x\tan^2 x\sec x + \sec x\tan x$

e) $\frac{d}{dx}\left(\frac{5\tan x + 3x^2\sin(x)}{(x^2+1)\sec x}\right)$

$= \frac{(x^2+1)\sec x\cdot\frac{d}{dx}(5\tan x + 3x^2\sin(x)) - (5\tan x + 3x^2\sin(x))\frac{d}{dx}(x^2+1)\sec x}{((x^2+1)\sec x)^2}$

$= \frac{1}{((x^2+1)\sec x)^2}\cdot\Big(\left(x^2+1\right)\sec x\cdot\left[5\sec^2 x + \frac{d}{dx}3x^2\sin(x)\right]$

$- (5\tan x + 3x^2\sin(x))\cdot\left[(x^2+1)\frac{d}{dx}\sec x + \sec x\frac{d}{dx}x^2+1\right]\Big)$

$= \frac{1}{((x^2+1)\sec x)^2}\cdot\Big((x^2+1)\sec x\cdot\left[5\sec^2 x + 3x^2\cos(x) + \sin x\cdot 6x\right]$

$- (5\tan x + 3x^2\sin(x))\cdot\left[(x^2+1)\sec x\tan x + \sec x\cdot 2x\right]\Big)$

(2) a) $\frac{d}{dx}\sin^2(x) = \frac{d}{dx}(\sin(x))^2 = 2\sin(x)\cdot\frac{d}{dx}\sin(x) = 2\sin(x)\cos(x)$

b) $\frac{d}{dx}\cos^2(x) = \frac{d}{dx}(\cos(x))^2 = 2\cos(x)\cdot\frac{d}{dx}\cos(x) = 2\cos(x)\cdot(-\sin(x))$

$= -2\cos(x)\sin(x)$

c) Using parts a) and b),

$\frac{d}{dx}(\sin^2(x)+\cos^2(x)) = 2\sin(x)\cos(x) - 2\cos(x)\sin(x) = 0$

This is not surprising, because $\sin^2 x + \cos^2 x = 1$,

so we expect its derivative to be zero.

(3) $s(t) = A\sin(\sqrt{k}\cdot t + B)$, where A, B, k are constants.

$s'(t) = A\cdot\cos(\sqrt{k}\cdot t + B)\cdot\frac{d}{dt}(\sqrt{k}\cdot t + B)$

$= A\cdot\cos(\sqrt{k}t + B)\cdot\sqrt{k}$

$= A\sqrt{k}\cos(\sqrt{k}t + B)$

$s''(t) = A\sqrt{k}\left(-\sin(\sqrt{k}\cdot t + B)\cdot\frac{d}{dt}(\sqrt{k}t + B)\right)$

$= -A\sqrt{k}\sin(\sqrt{k}t + B)\cdot\sqrt{k} = -k\cdot A\sin(\sqrt{k}t + B)$

$= -k\cdot s(t)$, so it satisfies $s''(t) = -k\,s(t)$

(4) derivatives of even functions:

$\frac{d}{dx}x^2 = 2x$, $\frac{d}{dx}x^4 = 4x^3$, $\frac{d}{dx}\cos(x) = -\sin(x)$

derivatives of odd functions:

$\frac{d}{dx}x^3 = 3x^2$, $\frac{d}{dx}x^5 = 4x^2$, $\frac{d}{dx}\sin(x) = \cos(x)$

In each case, the derivative of an even function is an odd function and the derivative of an odd function is an even function. I conjecture this is always true. Proof that the derivative of an even function is odd: Suppose $f(-x) = f(x)$ for all x. Taking the derivative of both sides, $\frac{d}{dx} f(-x) = \frac{d}{dx} f(x) = f'(x)$. Applying the chain rule to $\frac{d}{dx} f(-x)$ gives $f'(-x) \cdot \frac{d}{dx}(-x) = -f'(-x)$

So we see $f'(x) = -f'(-x)$, or $f'(-x) = -f'(x)$. This shows $f'(x)$ is odd. Similarly we can prove that the derivative of an odd function is even: Suppose $f(-x) = -f(x)$, then taking derivatives, $f'(-x) \cdot (-1) = -f'(x)$, and $f'(-x) = f'(x)$. So $f'(x)$ is even.

(5) $\frac{d}{dx} \sin(x^\circ) = \frac{d}{dx} \sin\left(\frac{\pi}{180} x\right) = \cos\left(\frac{\pi}{180} x\right) \frac{d}{dx}\left(\frac{\pi}{180} x\right)$

$$= \cos\left(\frac{\pi}{180} x\right) \cdot \frac{\pi}{180} = \frac{\pi}{180} \cos(x^\circ)$$

$$\frac{d}{dx} \cos(x^\circ) = \frac{d}{dx} \cos\left(\frac{\pi}{180} x\right) = -\sin\left(\frac{\pi}{180} x\right) \cdot \frac{d}{dx}\left(\frac{\pi}{180} x\right)$$

$$= -\sin\left(\frac{\pi}{180} x\right) \cdot \frac{\pi}{180} = -\frac{\pi}{180} \sin(x^\circ)$$

(6) $\lim_{x \to -2^-} f(x) = \lim_{x \to -2^-} 4 - x^2 = 0$, $\lim_{x \to -2^+} f(x) = \lim_{x \to -2^+} 4x = -8$

So $f(x)$ is <u>not</u> continuous at $x = -2$

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} 4x = 0$, $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} x^2 = 0$

So $f(x)$ <u>is</u> continuous at $x = 0$.

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} x^2 = 4$, $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} 4x - 4 = 4$

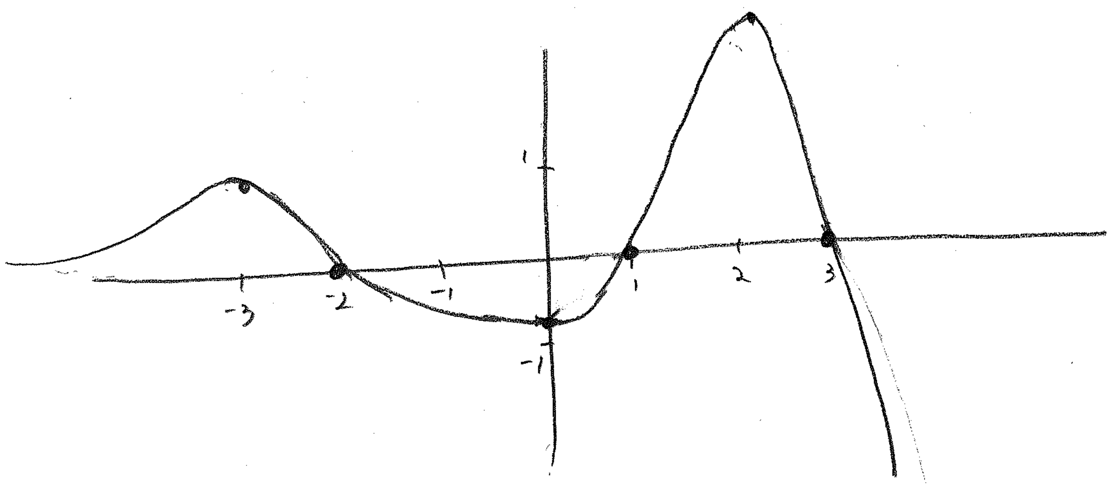
So $f(x)$ <u>is</u> continuous at $x = 2$

Since $f(x)$ is not continuous at $x = -2$, it <u>cannot</u> be differentiable there.

To test for differentiability at $x=0$, we need to check whether the slope of $4x$ at 0 matches the slope of x^2 at 0. This is NOT true since the slope on the left is 4 and on the right is 0. So f is not differentiable at $x=0$.

At $x=2$, the slope of x^2 at 2 is $2 \cdot 2$, or 4, and the slope of $4x-4$ at 2 is also 4. So the two slopes match, and $f(x)$ is differentiable at $x=2$.

(7)



The graph should show:

$f'(-2) = f'(1) = f(3) = 0$.

$f'(x)$ is positive for $x<-2$ and for $1<x<3$ and is negative for $-2<x<1$ and for $x>3$.

$f'(x)$ reaches maximums at about $x=-3$ and $x=2$, and a minimum at about 0.

As $x \to -\infty$ it looks like $f'(x) \to 0$.

As $x \to +\infty$ it looks like $f'(x) \to -\infty$.