Math 130-01, Spring 2020 Sample answers To Lab #7

① a) $\frac{d}{dx}\tan^{-1}(x) = (f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))} = \frac{1}{\sec^2(f^{-1}(x))} = \frac{1}{\sec^2(\tan^{-1}(x))}$

b) $\frac{\sin^2(\theta)}{\cos^2(\theta)} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta} \Rightarrow \left(\frac{\sin\theta}{\cos\theta}\right)^2 + 1 = \left(\frac{1}{\cos\theta}\right)^2$

$\Rightarrow \tan^2(\theta)+1 = \sec^2(\theta)$

c) Let $\theta = \tan^{-1}(x)$. Then $\sec^2(\theta) = \tan^2(\theta)+1$

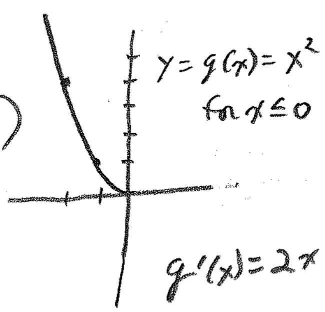
becomes $\sec^2(\tan^{-1}(x)) = (\tan(\tan^{-1}(x)))^2 + 1 = x^2+1$.

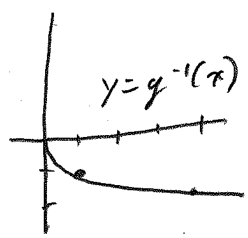
So $\frac{d}{dx}\tan^{-1}\theta = \frac{1}{x^2+1}$.

d) $(\tan(x))^{-1} = \frac{1}{\tan(x)} = \frac{1}{\frac{\sin(x)}{\cos(x)}} = \frac{\cos(x)}{\sin(x)} = \cot(x)$

$\frac{d}{dx}(\tan(x))^{-1} = \frac{d}{dx}\cot(x) = -\csc^2(x)$

② a)





$g'(x) = 2x$

b) $g^{-1}(x) = -\sqrt{x}$, The lower part of The graph of $y^2 = x$.

c) $(g^{-1})'(x) = \frac{1}{g'(g^{-1}(x))} = \frac{1}{2g^{-1}(x)} = \frac{1}{2\cdot(-\sqrt{x})} = -\frac{1}{2\sqrt{x}}$

③ a) If $f(x) = \frac{1}{x}$ and $f^{-1}(x) = \frac{1}{x}$, Then $f(f^{-1}(x)) = f\left(\frac{1}{x}\right) = \frac{1}{\frac{1}{x}} = x$, and similarly for $f^{-1}(f(x))$, So $f^{-1}\left(\frac{1}{x}\right)$ satisfies The definition.

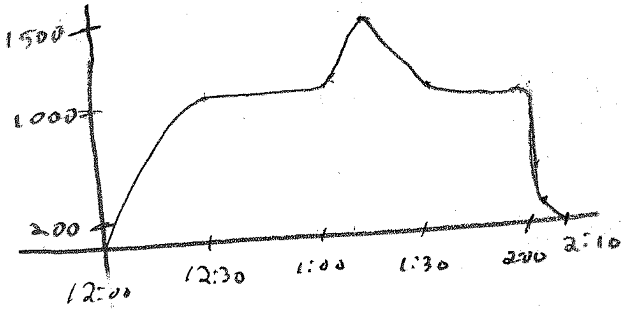
b) $\frac{d}{dx}\left(\frac{1}{x}\right) = (f^{-1})'(x) = \frac{1}{f'(f(x))} = \frac{1}{-\frac{1}{f(x)^2}} = -f(x)^2 = -\left(\frac{1}{x}\right)^2 = \frac{-1}{x^2}$

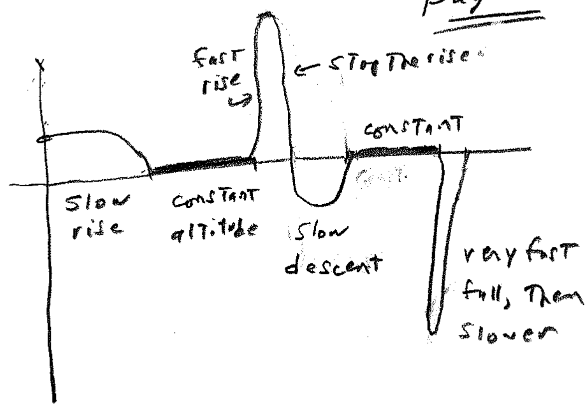
c) $g(g(x)) = g(\sqrt{1-x^2}) = \sqrt{1-(\sqrt{1-x^2})^2} = \sqrt{1-(1-x^2)} = \sqrt{x^2} = |x| = x$

$[\,|x| = x$ since $x > 0\,]$

So $g(x)$ is its own inverse!

④ a)





⑤ a) $\sin\left(\frac{\pi}{2}-x\right) = \sin\left(\frac{\pi}{2}\right)\cos(-x) + \cos\left(\frac{\pi}{2}\right)\sin(-x)$

$= 1 \cdot \cos(-x) + 0 \sin(-x)$

$= \cos(-x) = \cos(x)$

b) Plug $\frac{\pi}{2} - x$ in for x into $\sin\left(\frac{\pi}{2}-x\right) = \cos(x)$:

$\sin\left(\frac{\pi}{2} - \left(\frac{\pi}{2} - x\right)\right) = \cos\left(\frac{\pi}{2}-x\right) \Rightarrow \sin(x) = \cos\left(\frac{\pi}{2}-x\right)$

c) $\frac{d}{dx}\sin\left(\frac{\pi}{2}-x\right) = \cos\left(\frac{\pi}{2}-x\right)\cdot(-1) = -\cos\left(\frac{\pi}{2}-x\right) = -\sin(x) = \frac{d}{dx}\cos(x)$

⑥ a) $\frac{d}{dx}(x^2+x+1)\tan(x) = (x^2+x+1)\cdot\frac{d}{dx}\tan(x) + \tan(x)\cdot\frac{d}{dx}(x^2+x+1)$

$= (x^2+x+1)\cdot\sec^2(x) + \tan(x)\cdot(2x+1)$

b) $\frac{d}{dz}\frac{\sec^{-1}(x)}{3-\cos(x)} = \frac{(3-\cos(z))\frac{d}{dz}\sec^{-1}(z) - \sec^{-1}(z)\cdot\frac{d}{dz}(3-\cos(z))}{(3-\cos(z))^2}$

$= \frac{(3-\cos(z))\cdot\frac{1}{|z|\sqrt{z^2-1}} - \sec^{-1}(z)\cdot(\sin(z))}{(3-\cos(z))^2}$

c) $\frac{d}{dx}(\cos^{-1}(x))^5 = 5(\cos^{-1}(x))^4 \cdot \frac{-1}{\sqrt{1-x^2}}$

d) $\frac{d}{dx}\cot^{-1}(x\sec(x)) = \frac{-1}{1+(x\sec(x))^2}\cdot\left(x\cdot\sec(x)\tan(x) + \sec(x)\cdot 1\right)$

e) $\frac{d}{dx}x^2\sin^{-1}(x^2) = x^2\frac{d}{dx}\sin^{-1}(x^2) + \sin^{-1}(x^2)\cdot\frac{d}{dx}x^2$

$= x^2\cdot\frac{1}{\sqrt{1-(x^2)^2}}\cdot 2x + \sin^{-1}(x^2)\cdot 2x$