This lab is due in class on Friday. Remember to show your work and/or explain your reasoning for all problems on the lab! There will be a quiz in class tomorrow. And there's a test coming up Wednesday of next week. Next week's lab will be review for the test.

1. In class, we showed that $\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$. This problem asks you to show that $\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$.

a) Let $f(x) = \tan(x)$, for $-\frac{\pi}{2} < x < \frac{\pi}{2}$. Then $f(x)$ has an inverse function, which is written $f^{-1}(x) = \tan^{-1}(x)$. Apply the rule for the derivative of an inverse function to this function to find a formula for $\tan^{-1}(x)$. Your answer should use the function $\sec(x)$.

b) Starting from the basic identity $\sin^2(\theta) + \cos^2(\theta) = 1$, divide both sides by $\cos^2(\theta)$, and convert the resulting equation into an identity that relates $\tan(\theta)$ and $\sec(\theta)$.

c) Now use the results from parts **a)** and **b)** to rewrite the derivative of $\tan^{-1}(x)$ in the usual form. (Hint: Let $\theta = \tan^{-1}(x)$.)

d) As noted in class, the awful notation $\tan^{-1}(x)$ can lead to confusion because it does **not** mean $(\tan(x))^{-1}$. What basic trigonometric function is $(\tan(x))^{-1}$ equal to, and what is its derivative?

2. In class, we applied the rule for derivative of an inverse function to $f(x) = x^2$ for $x \geq 0$. Suppose that instead, we restrict the domain of x^2 to $x \leq 0$. That is, let $g(x) = x^2$ for $x \leq 0$. The function $g(x)$ has an inverse function.

a) Draw graphs of $g(x)$ and $g^{-1}(x)$.

b) Find a formula for the inverse function $g^{-1}(x)$.

c) Find a formula for its derivative, using the rule for the derivative of an inverse function. (Again, you should know what the answer is.)

3. The function $f(x) = \frac{1}{x}$ is unusual in that it is its own inverse. That is, its inverse function, $f^{-1}(x)$, is also equal to $\frac{1}{x}$.

a) Let $f(x) = \frac{1}{x}$. Show that its inverse function is $f^{-1}(x) = \frac{1}{x}$. (Recall the definition of inverse function: $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.)

b) Apply the rule for the derivative of an inverse function to $f(x) = \frac{1}{x}$ and $f^{-1}(x) = \frac{1}{x}$. You know what the answer should be; make sure that you get it!

c) Let $g(x) = \sqrt{1-x^2}$ for $0 < x < 1$. Show that $g(x)$ is its own inverse.

4. For review: A hot air balloon takes off at 12:00 noon and rises slowly until it reaches an altitude of 1000 feet at 12:30. It stays at that altitude for thirty minutes, but then encounters an updraft that lifts it rapidly to 1500 feet. But the balloonist manages to stop the rise and returns more slowly back to an altitude of 1000 feet at 1:30. It stays at that altitude until at 2:00 — but then a problem with the balloon causes it to plunge down to 200 feet in just a few minutes! Fortunately, at that time, the balloonist saves the day, recovers control, and drops slowly back to the ground at 2:10.

a) Draw a graph of altitude versus time that illustrates this story.

b) Now, draw a graph of the derivative of altitude with respect to time that agrees with the story. (The graph will, of course, be even more approximate than the graph for part **a)**.)

5. In this problem, you will look at the trigonometric identities $\cos(x) = \sin\left(\frac{\pi}{2} - x\right)$ and $\sin(x) = \cos\left(\frac{\pi}{2} - x\right)$. You will need the fact that $\cos(-x) = \cos(x)$, and you will need the angle sum formula

$$\sin(A + B) = \sin(A)\cos(B) + \sin(B)\cos(A)$$

a) Use the angle sum formula to show that $\sin\left(\frac{\pi}{2} - x\right) = \cos(x)$.

b) Show that $\sin(x) = \cos\left(\frac{\pi}{2} - x\right)$, without using an angle sum formula. (Do some simple math on the formula from part **a)**.)

c) If $\cos(x)$ and $\sin\left(\frac{\pi}{2} - x\right)$ are the same function, then they should have the same derivative. Verify that that is the case.

6. Compute the following derivatives. You can use all of the differentiation rules that we have covered, including the derivatives of the trigonometric functions, which are given here for reference.

$\frac{d}{dx}\sin(x) = \cos(x)$	$\frac{d}{dx}\cos(x) = -\sin(x)$	$\frac{d}{dx}\tan(x) = \sec^2(x)$
$\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$	$\frac{d}{dx}\csc(x) = -\csc(x)cot(x)$	$\frac{d}{dx}\cot(x) = -\csc^2(x)$
$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}\cos^{-1}(x) = \frac{-1}{\sqrt{1-x^2}}$	$\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$
$\frac{d}{dx}\sec^{-1}(x) = \frac{1}{\lvert x\rvert\sqrt{x^2-1}}$	$\frac{d}{dx}\csc^{-1}(x) = \frac{-1}{\lvert x\rvert\sqrt{x^2-1}}$	$\frac{d}{dx}\cot^{-1}(x) = \frac{-1}{1+x^2}$

a) $\frac{d}{dx}\left(x^2 + x + 1\right)\tan^{-1}(x)$

b) $\frac{d}{dz}\frac{\sec^{-1}(z)}{3 - \cos(z)}$

c) $\frac{d}{dx}\left(\cos^{-1}(x)\right)^5$

d) $\frac{d}{dx}\cot^{-1}(x\sec(x))$

e) $\frac{d}{dx}x^2\sin^{-1}(x^2)$