Math 130-01 Sample Answers for Lab #8

(1) a) $x = -4, -2, 1$ (where tangent line is horizontal)

b) $-2 < x < 1$ (where tangent line slopes upward)

c) $f'(-1)$ (it looks like tangent line is steeper than at $x=0$)

d) -1, since the graph looks like $x=-y$ for $x>3$

(2) $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} 3x^2 = 3$; $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} x^3+2 = 3$. Since these limits both equal $f(1)$, $f(x)$ is continuous at $x=1$. Since $3x^2$ has slope 6 while x^3+2 has slope 3 at $x=1$, the slopes do not match, and $f(x)$ is not differentiable at 1.

(3) $v(t) = s'(t) = 10\left(0 - e^{-t/5}\cdot \frac{d}{dt}(-t/5)\right) = 10\cdot(-e^{-t/5})(-\frac{1}{5}) = 2e^{-t/5}$

$a(t) = v'(t) = 2\cdot e^{-t/5}\cdot \frac{d}{dt} e^{-t/5} = -\frac{2}{5}e^{-t/5}$. The point is moving right since its velocity is always positive.

(4) To show (e,e) is on the graph, check that $e = a^e$: $a^e = (e^{1/e})^e = e^{(1/e\cdot e)} = e^1 = e$. So, (e,e) is on the graph. The derivative of a^x is $a^x \ln(a)$, so the slope of the tangent line at $x=e$ is $a^e\cdot \ln(a) = (e^{1/e})^e\cdot \ln(e^{1/e}) = e^1\cdot 1/e = 1$. The tangent line has slope 1 and contains (e,e) so has equation $y-e = 1\cdot(x-e)$, that is, $y=x$.

(5) $\frac{d}{dx} e^{2x} = e^{2x}\frac{d}{dx}(2x) = 2e^{2x}$. $\frac{d^2}{dx^2} e^{2x} = \frac{d}{dx}\left(2e^{2x}\right) = 2\cdot e^{2x}\cdot 2 = 2^2 e^{2x}$. $\frac{d^3}{dx^3} e^{2x} = \frac{d}{dx} 2^2 e^{2x} = 2^3 e^{2x}$. It is clear that in general, $\frac{d^n}{dx^n} e^{2x} = 2^n e^{2x}$.

(6) $F''(x) = \frac{d}{dx} F'(x) = \frac{d}{dx} e^{-x^2} = e^{-x^2}\cdot \frac{d}{dx}(-x^2) = -2xe^{-x^2}$.

$r'(x) = F'(3x^2+1)\cdot \frac{d}{dx}(3x^2+1) = F'(3x^2+1)\cdot 6x$

$= e^{-(3x^2+1)^2}\cdot 6x$. $r'(1) = e^{-4^2}\cdot 6 = 6\cdot e^{-16}$

(7) An inverse function for a function $f(x)$ is a function $g(x)$ such that $f(g(x)) = x$ for all x in the domain of g, and $g(f(x)) = x$ for all x in the domain of f. For example $\ln(x)$ is an inverse function for e^x because $\ln(e^x) = x$ and $e^{\ln(x)} = x$ for $x>0$.

(8) When a quantity B is determined by a quantity A, we can ask about the rate at which B changes as A changes. An average rate of change would be given by $\Delta B/\Delta A$, where ΔA represents some change in the value of A and ΔB represents the resulting change in the value of B. Taking a limit as $\Delta A \to 0$ gives the derivative dB/dA as an exact rate of change. [Often, A represents time]

(9) a) $\frac{d}{dx}(2x^5 - 3x^7 + 4) = 2\cdot 5x^4 - 3\cdot 7x^6 + 0 = 10x^4 - 21x^6$

b) $\frac{d}{dx}(3x^{7/3} + 7x^{3/7}) = 3\cdot\frac{7}{3}x^{7/3-1} + 7\cdot\frac{3}{7}\cdot x^{3/7-1} = 7x^{4/3} - 3x^{-4/7}$

c) $$\frac{d}{dt}\left(\frac{\sin(2t)-\cos(3t)}{t+e^t}\right) = \frac{(t+e^t)\cdot\frac{d}{dt}(\sin(2t)-\cos(3t)) - (\sin(2t)-\cos(3t))\frac{d}{dt}(t+e^t)}{(t+e^t)^2}$$

$$= \frac{(t+e^t)(\cos(2t)\cdot 2 + \sin(3t)\cdot 3) - (\sin(2t)-\cos(3t))\cdot(1+e^t)}{(t+e^t)^2}$$

d) $\frac{d}{dx} e^{2x}\cos(e^x) = e^{2x}\frac{d}{dx}\cos(e^x) + \cos(e^x)\cdot\frac{d}{dx}e^{2x}$

$= e^{2x}(-\sin(e^x)e^x) + \cos(e^x)\cdot e^{2x}\cdot 2$

e) $\frac{d}{d\theta}(\theta^3 + \theta\sin\theta) = \frac{d}{d\theta}(\theta^3) + \frac{d}{d\theta}(\theta\sin\theta)$

$= 3\theta^2 + \theta\cdot\frac{d}{d\theta}\sin\theta + \sin\theta\cdot\frac{d}{d\theta}\theta$

$= 3\theta^2 + \theta\cos\theta + \sin\theta$

f) $\frac{d}{dy}\left(\ln(y^2+1)\right)^{100} = 100\left(\ln(y^2+1)\right)^{99} \cdot \frac{d}{dy}\ln(y^2+1)$

$= 100 \cdot \left(\ln(y^2+1)\right)^{99} \cdot \frac{1}{y^2+1} \cdot 2y$

10 a) $\frac{d}{dx}\left(\frac{1}{x} - \tan^{-1}(x)\right) = \frac{d}{dx}\left(\frac{1}{x}\right) - \frac{d}{dx}\tan^{-1}(x) = -\frac{1}{x^2} - \frac{1}{1+x^2}$

b) $\frac{d}{dt}\left(\tan(t^2) + t^2\cot(t)\right) = \frac{d}{dt}\tan(t^2) + \frac{d}{dt}\left(t^2\cot(t)\right)$

$= \sec^2(t^2) \cdot \frac{d}{dt}t^2 + t^2\frac{d}{dt}\cot(t) + \cot(t) \cdot \frac{d}{dt}(t^2)$

$= \sec^2(t^2) \cdot 2t + t^2(-\csc^2(t)) + \cot(t) \cdot 2t$

c) $\frac{d}{dx}\sin^{-1}(2^x) = \frac{1}{\sqrt{1-(2^x)^2}} \cdot \frac{d}{dx}2^x$

$= \frac{1}{\sqrt{1-2^{2x}}} \cdot 2^x \cdot \ln(2)$

11 $\frac{d}{dx} e^{\pi} \cdot \pi^x \cdot x^{\pi} = e^{\pi} \cdot \frac{d}{dx}\left(\pi^x \cdot x^{\pi}\right)$ [e^{π} is a constant!]

$= e^{\pi} \cdot \left(\pi^x \frac{d}{dx}x^{\pi} + x^{\pi}\frac{d}{dx}\pi^x\right) = e^{\pi}\left(\pi^x \cdot \pi x^{\pi-1} + x^{\pi} \cdot \pi^x \ln(\pi)\right)$

12 The limit is $f'(2)$ where $f(x) = \sqrt{5-x^2}$.

$f'(x) = \frac{1}{2\sqrt{5-x^2}} \cdot \frac{d}{dx}(5-x^2) = \frac{-x}{\sqrt{5-x^2}}$, and

$f'(2) = \frac{-2}{\sqrt{5-2^2}} = \frac{-2}{\sqrt{1}} = -2.$

13 $\sin(\sin^{-1}(x)) = x$, and $\cos(\sin^{-1}(x)) =$

$\sqrt{1-\sin^2(\sin^{-1}(x))} = \sqrt{1-x^2}$. So

$\sin(\sin^{-1}(x)) + \cos(\sin^{-1}(x)) = x + \sqrt{1-x^2}.$

14 a) $\frac{d}{dx}(xe^{-y} + \sin(xz)) = \frac{d}{dx}xe^{-y} + \frac{d}{dx}\sin(xz)$

$= e^{-y}\frac{d}{dx}x + \cos(xz)\frac{d}{dx}xz = e^{-y} + \cos(xz) \cdot z$

b) $\frac{d}{dy}(xe^{-y} + \sin(xz)) = \frac{d}{dy}xe^{-y} + \frac{d}{dy}\sin(xz)$

$= x \cdot \frac{d}{dx}e^{-y} + 0 = x \cdot e^{-y}\frac{d}{dy}(-y) = -xe^{-y}$

c) $\frac{d}{dz}(xe^{-y} + \sin(xz)) = \frac{d}{dz}xe^{-y} + \frac{d}{dz}\sin(xz)$

$= 0 + \cos(xz)\frac{d}{dz}xz = \cos(xz) \cdot x$

d) $\frac{d}{dt}(xe^{-y} + \sin(xz)) = 0$

15 a) $\frac{d}{dx}\ln(ax+b) = \frac{1}{ax+b} \cdot \frac{d}{dx}(ax+b) = \frac{a}{ax+b}$

b) $\frac{d}{dx}(f(x))^n = n(f(x))^{n-1} \cdot \frac{d}{dx}f(x) = n(f(x))^{n-1} \cdot f'(x)$

c) $\frac{d}{dx}f(e^x) = f'(e^x) \cdot \frac{d}{dx}e^x = f'(e^x)e^x = e^x f'(e^x)$