Math 131-03, Spring 2017 — Final Exam Info

The final exam for this course is scheduled for Tuesday, May 9, at 1:30 PM, in our regular classroom. The exam includes everthing that we have done in the course, with some emphasis on the material covered since the third test. Although the exam period is three hours long, most people should finish in less than two hours. The exam will be six or seven pages long, with the usual mix of problems, definitions, and essays. There will be at least one longer essay summarizing some central ideas in the Calculus.

As usual, you will not need a calculator for the test. A basic non-graphing calculator will be provided to you, and you will be permitted to use only the calculator that is provided. Scrap paper will also be provided. All you need is a pencil or pen.

Here are some terms and ideas covered after the third test:

additional tests for convergence of series:
- The Comparison Test
- The Ratio Test
- The Limit Comparison Test
- The Root Test
- The Alternating Series Test

absolute convergence of series

conditional convergence of series

if a series converges absolutely, then it converges

alternating series

the remainder in a convergent alternating series

power series

radius of convergence of a power series

interval of convergence of a power series

using the ratio (or root) test to find the radius of convergence

integration and differentiation of power series

if $f(x) = \sum_{n=0}^{\infty} c_n(x-a)^n$ on an interval around a, then $c_n = \frac{f^{(n)}(a)}{n!}$

Taylor series centered at a for a function $f(x)$

the Taylor series for a function does **not** converge to that function in all cases

Maclaurin series (just a Taylor series at 0)

Taylor polynomial of order n, centered at a, for a function $f(x)$

Here are some major terms and idea from earlier in the semester:

Riemann sum; left, right, and midpoint Riemann sums

area under a curve

definite integral, defined as a limit of Riemann sums

properties of definite integrals

how definite integrals relate to area

the Fundamental Theorem of Calculus, part I and part II

method of substitution ("change of variables") for finding an indefinite integral

net change of a quantity $Q(t)$, given by $\int_a^b Q'(t)dt$

application to velocity, acceleration, displacement, distance traveled

area between curves

volume; volumes by slicing: integrating the cross-sectional area

volumes of revolution; the disk method, the washer method, and the shell method

integrating with respect to y instead of with respect to x

length of a curve

change of variables in a definite integral

integration by parts

partial fractions

differential equations and initial value problems

solving separable first-order differential equations

improper integrals; convergence and divergence of improper integrals

infinite sequences and infinite series

partial sums of an infinite series

convergence and divergence of sequences and series

properties of sequences and series

geometric sequences and geometris series

the Divergence Test

the Integral Test

p-series

Note: See also the study guides for the three tests, which you can find on-line.

Tests for convergence of infinite series...

Geometric Series Test: A geometric series with ratio r diverges if $|r| \geq 1$ and converges if $|r| < 1$. A convergent geometric series converges to $\frac{a}{1-r}$ where a is the initial term and r is the ratio.

Divergence Test (also called the n^{th} term test): If $\lim_{n\to\infty} a_n \neq 0$ (including the case $\lim_{n\to\infty} a_n$ does not exist) then the infinite series $\sum_{n=0}^{\infty} a_n$ diverges.

Integral Test: Suppose that $f(x)$ is a positive, decreasing function for $x \geq 1$, and suppose $a_n = f(n)$ for $n = 1, 2, 3, \ldots$. If the improper integral $\int_1^{\infty} f(x)\,dx$ converges, then $\sum_{n=1}^{\infty} a_n$ also converges. If the improper integral $\int_1^{\infty} f(x)\,dx$ diverges, then $\sum_{n=1}^{\infty} a_n$ also diverges.

p-series: $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \leq 1$.

Alternating Series Test: Suppose that $\{a_n\}_{n=0}^{\infty}$ is a decreasing sequence of positive terms such that $\lim_{n\to\infty} a_n = 0$. Then $\sum_{n=0}^{\infty} a_n$ converges.

Comparison Test: Suppose that $\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} b_n$ are infinite series of positive terms and that $a_n \leq b_n$ for all n. If $\sum_{n=0}^{\infty} b_n$ converges, then $\sum_{n=0}^{\infty} a_n$ also converges. If $\sum_{n=0}^{\infty} a_n$ diverges, then $\sum_{n=0}^{\infty} b_n$ also diverges.

Limit Comparison Test: Suppose that $\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} b_n$ are infinite series of positive terms and that $0 < \lim_{n\to\infty} \frac{a_n}{b_n} < \infty$. Then $\sum_{n=0}^{\infty} a_n$ converges if and only if $\sum_{n=0}^{\infty} b_n$ converges.

Ratio Test: Suppose that $\sum_{n=0}^{\infty} a_n$ is an infinite series of positive terms. Let $L = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$. Assume the limit exists or is infinite. If $L < 1$, then the series converges. If $L > 1$ (including $L = \infty$), then the series diverges. (No information if $L = 1$.)

Ratio Test: Suppose that $\sum_{n=0}^{\infty} a_n$ is an infinite series of positive terms, and let $L = \lim_{n\to\infty} (a_n)^{1/n}$. Assume the limit exists or is infinite. If $L < 1$, then the series converges. If $L > 1$ (including $L = \infty$), then the series diverges. (No information if $L = 1$.)

Note that the last four tests can be used to test for absolute convergence of any series, by taking the absolute value of the terms from the series.

Most of the final exam from Math 131 in Spring 2011:

1. For each of the following infinite series, determine whether the series *converges* or *diverges*. State clearly which rule you are applying and why it applies.

a) $\displaystyle\sum_{k=1}^{\infty} \frac{k^3}{k^6+4}$ **b)** $\displaystyle\sum_{k=1}^{\infty} \frac{k!}{7^k\sqrt{k}}$ **c)** $\displaystyle\sum_{k=1}^{\infty} \left(-\frac{3}{5}\right)^{k+1}$ **d)** $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k k^2}{k^2+1}$

2. Suppose that $g(x)$ is a function and that $g(x) = \displaystyle\sum_{k=1}^{\infty} \frac{x^k}{k \cdot 5^k}$. Find a series that converges to $g'(x)$, and then find $g'(3)$.

3. Let $f(x) = \sqrt{x}$. Find the *Taylor polynomial* of degree 3, centered at $a = 1$, for $f(x)$.

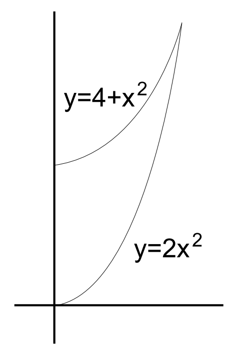
4. Suppose that $\{a_k\}_{k=1}^{\infty}$ is a sequence of positive numbers ($a_k > 0$), and that the series $\displaystyle\sum_{k=0}^{\infty} a_k$ converges. Show that $\displaystyle\sum_{k=0}^{\infty} a_k^2$ also converges. (Hint: $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$.)

5. Find each of the following indefinite integrals.

a) $\displaystyle\int x\sin(2x^2+1)\,dx$ **b)** $\displaystyle\int x\sin(2x+1)\,dx$

c) $\displaystyle\int \frac{2x\cos(x^2)}{\sin(x^2)+2}\,dx$ **d)** $\displaystyle\int \frac{5}{(x-3)(2x-1)}\,dx$

6. Consider the region in the first quadrant bounded by the y-axis and by the curves $y = 4 + x^2$ and $y = 2x^2$.



a) Write a *Left Riemann Sum*, using 4 subintervals, to approximate the area of the region.

b) Use an integral to find the exact area of the region.

c) Write but **do not evaluate** an integral that gives the volume of the sold that is generated when the region is rotated about the y-axis.

d) Write but **do not evaluate** an integral that gives the volume of the sold that is generated when the region is rotated about the x-axis.

e) Write but **do not evaluate** an integral that gives the volume of the sold that is generated when the region is rotated about the vertical line $y = -2$.

7. A point moves along a line. It starts with velocity zero at time $t = 0$. It accelerates with an acceleration given by $a(t) = e^{-t}$ feet per second per second. How far has it moved at time $t = 2$ seconds?

8. The *Fundamental Theorem of Calculus* shows the relationship between antiderivatives and definite integrals. This theorem has two parts. Choose *either* part, state it, and explain its importance.

9. Throughout two terms of calculus, you have used *limits* in various ways. In each case, limits bring clarity and rigor to a problem that would be hard to deal with in any other way. Write an essay that describes some of the applications of limits in calculus. In each case, discuss the problem that is solved by using limits, and explain why limits are so essential to the solution.