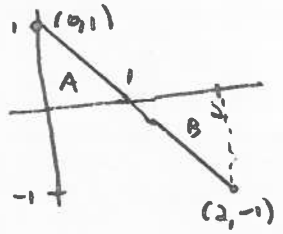
Math 131-03, Spring 2017 Sample Answers to Homework # 2

Section 5.2

(28)



The graph of $y = 1-x$ is a line that goes through the points $(0,1)$, $(1,0)$, and $(2,-1)$. $\int_0^2 1-x\,dx$ can be computed as the area of region A minus the area of region B. Each region is a triangle that has area $\frac{1}{2}$. So $\int_0^2 1-x\,dx = \frac{1}{2} - \frac{1}{2} = \underline{\underline{0}}$.

(44) a) $\int_0^5 f(x)\,dx = \int_0^2 f(x)\,dx + \int_2^5 f(x)\,dx = 6 + (-8) = \underline{\underline{-2}}$

b) $\int_0^5 |f(x)|\,dx = \int_0^2 |f(x)|\,dx + \int_2^5 |f(x)|\,dx$

$= \int_0^2 f(x)\,dx + \left(-\int_2^5 f(x)\,dx\right)$, since $f(x) \geq 0$ on $[0,2]$ and $f(x) \leq 0$ on $[2,5]$

$= 6 + (-(-8)) = \underline{\underline{14}}$

c) $\int_2^5 4|f(x)|\,dx = 4 \cdot \int_2^5 |f(x)|\,dx = 4 \cdot 8 = \underline{\underline{32}}$.

d) $\int_0^5 \left(f(x) + |f(x)|\right)dx = \int_0^5 f(x)\,dx + \int_0^5 |f(x)|\,dx$

$= -2 + 14$, from parts a) and b)

$= \underline{\underline{12}}$

(50) $\int_0^2 (x^2-1)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k^*)\Delta x$, where $\Delta x = \frac{2}{n}$, $x_k^* = 0 + k\Delta x = \frac{2k}{n}$

$$= \lim_{n\to\infty} \sum_{k=1}^{n} f\left(\frac{2k}{n}\right)\cdot\frac{2}{n} = \lim_{n\to\infty} \sum_{k=1}^{n} \left(\left(\frac{2k}{n}\right)^2 - 1\right)\cdot\frac{2}{n}$$

$$= \lim_{n\to\infty} \left(\sum_{k=1}^{n} \left(\frac{2k}{n}\right)^2\cdot\frac{2}{n} - \sum_{k=1}^{n} \frac{2}{n}\right) = \lim_{n\to\infty} \left(\sum_{k=1}^{n} \frac{8k^2}{n^3} - \sum_{k=1}^{n} \frac{2}{n}\right)$$

$$= \lim_{n\to\infty} \left(\frac{8}{n^3} \sum_{k=1}^{n} k^2 - \frac{2}{n} \sum_{k=1}^{n} 1\right)$$

$$= \lim_{n\to\infty} \left(\frac{8\cdot(n\cdot(n+1)\cdot(2n+1))}{n^3\cdot 6} - \frac{2}{n}\cdot n\right)$$

$$= \lim_{n\to\infty} \left(\frac{16n^3 + 24n^2 + 8n}{6n^3} - 2\right) = \frac{16}{6} - 2 = \frac{8}{3} - 2 = \underline{\underline{\frac{2}{3}}}$$

Section 5.3

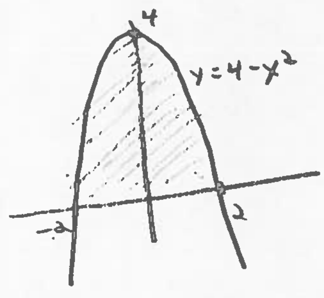
(30) $\int_0^2 (3x^2+2x)\,dx = x^3+x^2 \Big|_0^2 = (2^3+2^2)-0 = \underline{\underline{12}}$

(46) $\int_4^9 \frac{x-\sqrt{x}}{x^3}\,dx = \int_4^9 \frac{x}{x^3} - \frac{x^{1/2}}{x^3}\,dx = \int_4^9 x^{-2} - x^{-5/2}\,dx$

$= \frac{x^{-1}}{-1} - \frac{x^{-3/2}}{-3/2}\Big|_4^9 = -\frac{1}{x} + \frac{2}{3}\cdot\frac{1}{x^{3/2}}\Big|_4^9$

$= \left(-\frac{1}{9}+\frac{2}{3}\cdot\frac{1}{27}\right) - \left(-\frac{1}{4}+\frac{2}{3}\cdot\frac{1}{8}\right) = \underline{\underline{-\frac{1}{9}+\frac{2}{81}+\frac{1}{4}-\frac{1}{12}}} = \frac{13}{162}$

(52)



The graph of $f(x) = 4-x^2$ intersects the x-axis at $x=-2$ and $x=2$, as shown. The area in question is the shaded region, which is given by the definite integral $\int_{-2}^2 4-x^2\,dx$.

$\int_{-2}^2 4-x^2\,dx = 4x - \frac{x^3}{3}\Big|_{-2}^2 = \left(8-\frac{8}{3}\right) - \left(-8-\left(\frac{-8}{3}\right)\right)$

$= 8-\frac{8}{3}+8-\frac{8}{3} = 16 - \cancel{16/3} = \underline{\underline{\frac{32}{3}}}$

(62) $\frac{d}{dx}\int_0^x e^t\,dt = \underline{\underline{e^x}}$, by a direct application of the Fundamental Theorem of Calculus, part 1

(64) $\int_{x^2}^{10} \frac{dz}{z^2+1} = -\int_{10}^{x^2} \frac{dz}{z^2+1} = -\int_{10}^{x^2} \frac{1}{z^2+1}\,dz$.

We can find the derivative by using the Fundamental Theorem of Calculus together with the chain rule:

$\frac{d}{dx}\left(\int_{x^2}^{10} \frac{dz}{z^2+1}\right) = -\frac{d}{dx}\int_{10}^{x^2}\frac{1}{z^2+1}\,dz = -\frac{1}{(x^2)^2+1}\cdot\frac{d}{dx}(x^2)$

$= -\frac{1}{x^4+1}\cdot 2x = \underline{\underline{\frac{-2x}{x^4+1}}}$