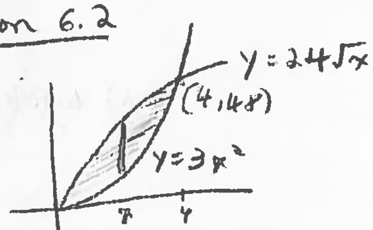
Section 6.2

(14)



$A = \int_a^b f(x) - g(x)\, dx$, where $f(x)$ is the top function, $g(x)$ is the bottom.

$= \int_0^4 24\sqrt{x} - 3x^2\, dx$

$= 24 \cdot \frac{2}{3} x^{3/2} - x^3 \Big|_0^4$

$= 16x^{3/2} - x^3$

$= (16 \cdot 4^{3/2} - 4^3) - (0)$

$= 16 \cdot 8 - 64 = 128 - 64 = \underline{\underline{64}}$

find intersections:

$24\sqrt{x} = 3x^2 \Rightarrow 8\sqrt{x} = x^2$

$\Rightarrow 64x = x^4$

$\Rightarrow x = 0$ or $64 = x^3$

$\Rightarrow x = 0$ or $x = 4$

intersections at $(0,0), (4, 48)$

(26) intersections at $y = 0, 1, 4$
[shown by graph, and checked by plugging in values]

$A = \int_a^b (\text{right} - \text{left})\, dy$,

but the functions cross at $x = 1$

$\int_0^1 (y^3 - 4y^2 + 3y) - (y^2 - y)\, dy + \int_1^4 (y^2 - y) - (y^3 - 4y^2 + 3y)\, dy$

$= \int_0^1 y^3 - 5y^2 + 4y\, dy + \int_1^4 5y^2 - y^3 - 4y\, dy$

$= \left(\frac{y^4}{4} - \frac{5y^3}{3} + 2y^2 \Big|_0^1 \right) + \left(\frac{5y^3}{3} - \frac{y^4}{4} - 2y^2 \Big|_1^4 \right)$

$= \left(\frac{1}{4} - \frac{5}{3} + 2 \right) + \left[\left(\frac{5 \cdot 64}{3} - 64 - 32 \right) - \left(\frac{5}{3} - \frac{1}{4} - 2 \right) \right]$

$= \underline{\underline{\frac{71}{6}}}$

(28) intersections:

$x^2 - 4x = 2x - 8$

$x^2 - 6x + 8 = 0$

$(x-2)(x-4) = 0$

$x = 2, x = 4$

intersections at $(2,-4), (4,0)$

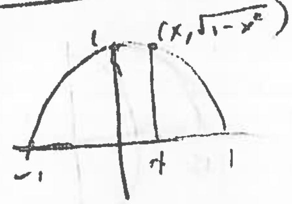
w.r.t x: $\int_0^2 0 - (x^2 - 4x)\, dx + \int_2^4 0 - (2x - 8)\, dx$

w.r.t. y: $\int_{-4}^0 \left(\frac{1}{2}y + 4\right) - \left(\frac{4 - \sqrt{16 + 4y}}{2} \right) dy$

solve for y in terms of x:
$\begin{cases} y = 2x - 8 \Rightarrow y + 8 = 2x \Rightarrow \frac{1}{2}y + 4 = x \text{ or } x = \frac{1}{2}y + 4 \\ y = x^2 - 4x \Rightarrow x^2 - 4x - y = 0 \Rightarrow y = \frac{4 \pm \sqrt{16 + 4y}}{2}. \end{cases}$ we need the negative $\sqrt{\ }$.

Section 6.3

(8)



$A(x) = \left(\sqrt{1 - x^2}\right)^2$

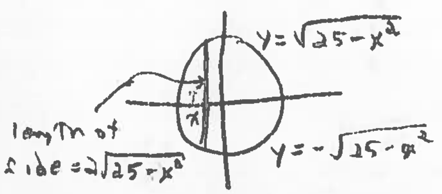
$V = \int_{-1}^1 A(x)\, dx$

$= \int_{-1}^1 \left(\sqrt{1 - x^2}\right)^2 dx = \int_{-1}^1 1 - x^2\, dx$

$= x - \frac{x^3}{3} \Big|_{-1}^1 = \left(1 - \frac{1}{3}\right) - \left(-1 + \frac{1}{3}\right)$

$= \underline{\underline{\frac{4}{3}}}$

(10) The area of an equilateral Triangle with side s is $\frac{\sqrt{3}}{4}s^2$.



$A(x) = \frac{\sqrt{3}}{4}\left(2\sqrt{25-x^2}\right)^2 = \sqrt{3}(25-x^2)$

$V = \int_{-5}^{5} A(x)\,dx$

$= \int_{-5}^{5} \sqrt{3}(25-x^2)\,dx$

$= \sqrt{3}\left(25x - \frac{x^3}{3}\Big|_{-5}^{5}\right)$

$= \sqrt{3}\left(\left(125 - \frac{125}{3}\right) - \left(-125 + \frac{125}{3}\right)\right)$

$= \sqrt{3}\cdot\left(\frac{250}{3} - \left(-\frac{250}{3}\right)\right) = \underline{\underline{\frac{\sqrt{3}}{3}\cdot 500}}$

(20) $V = \int_0^{\pi/2} \pi\cdot(f(x))^2\,dx = \int_0^{\pi/2} \pi\cdot(\cos x)^2\,dx = \pi\int_0^{\pi/2} \cos^2 x\,dx$

$= \pi\int_0^{\pi/2} \frac{1}{2}(1+\cos 2x)\,dx = \frac{\pi}{2}\cdot\left(x + \frac{1}{2}\sin(2x)\Big|_0^{\pi/2}\right)$

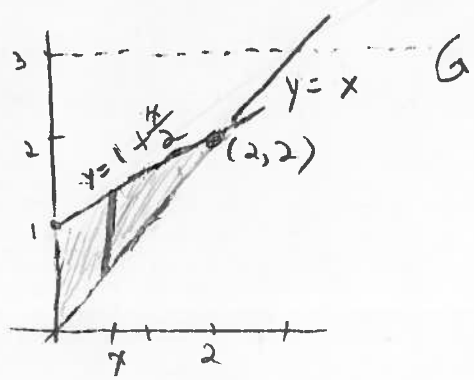
$= \frac{\pi}{2}\left[\left(\frac{\pi}{2} + \frac{1}{2}\sin\pi\right) - \left(0 + \frac{1}{2}\sin 0\right)\right] = \frac{\pi}{2}\left[\left(\frac{\pi}{2} - 0\right) - (0)\right] = \underline{\underline{\frac{\pi^2}{4}}}$

(36) $y = \ln(x) \Rightarrow x = e^y$

$V = \int_0^2 \pi(f(y))^2\,dy = \int_0^2 \pi(e^x)^2\,dx = \pi\int_0^2 e^{2x}\,dx = \pi\cdot\frac{1}{2}e^{2x}\Big|_0^2$

$= \pi\cdot\left(\frac{1}{2}e^4 - \frac{1}{2}e^0\right) = \underline{\underline{\frac{\pi}{2}\left(e^4 - 1\right)}}$

(50)



outer radius =

$3 - x$

inner radius =

$3 - \left(1 + \frac{x}{2}\right) = 2 - \frac{x}{2}$

$V = \int_0^2 \pi\left(R_1^2 - R_2^2\right)\,dx$

$= \int_0^2 \pi\left((3-x)^2 - \left(2 - \frac{x}{2}\right)^2\right)\,dx$

$= \pi\int_0^2 (9 - 6x + x^2) - \left(4 - 2x + \frac{x^2}{4}\right)\,dx$

$= \pi\int_0^2 5 - 4x + \frac{3}{4}x^2\,dx$

$= \pi\left(5x - 2x^2 + \frac{1}{4}x^3\Big|_0^2\right)$

$= \pi\left(\left(10 - 8 + \frac{8}{4}\right) - (0)\right)$

$= \underline{\underline{4\pi}}$