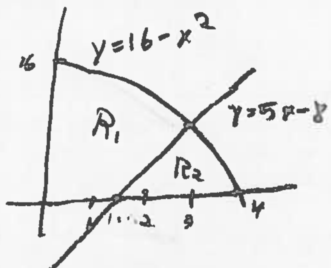
Chapter 6 Review

(14)



intersection:

$16-x^2 = 5x-8$

$0 = x^2+5x-24$

$= (x+8)(x-3)$

so $\boxed{x=3}$, or $x=-8$

intersection at $(3,7)$

Area of $R_1 + R_2$ is $\int_0^4 16-x^2\,dx$

$$= 16x - \frac{x^3}{3}\Big|_0^4 = 16\cdot 4 - \frac{64}{3} = 64 - \frac{64}{3} = \frac{128}{3}$$

$$\text{Area of } R_2 = \int_{8/5}^{3} 5x-8\,dx + \int_3^4 16-x^2\,dx$$

$$= \frac{1}{2}\cdot\left(3-\frac{8}{5}\right)\cdot 7 + \left(16x - \frac{x^3}{3}\Big|_3^4\right)$$

[geometrically]

$$= \frac{49}{10} + \left(\frac{128}{3} - \left(48 - \frac{27}{3}\right)\right)$$

$$= \frac{49}{10} + \frac{128}{3} - 39 = \underline{\underline{\frac{257}{30}}} \approx 8.5667$$

$$\text{Area of } R_1 = \frac{128}{3} - \frac{257}{30} = \underline{\underline{\frac{1023}{30}}} = \frac{341}{10} = 34.1$$

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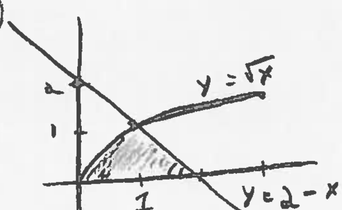

intersection at $(1,1)$

$$A(x) = \frac{\pi R^2}{2} = \begin{cases} \frac{\pi}{2}\left(\frac{1}{2}\sqrt{x}\right)^2, & 0 \le x \le 1 \quad (R = \frac{\sqrt{x}}{2}) \\ \frac{\pi}{2}\left(\frac{1}{2}(2-x)\right)^2, & 1 \le x \le 2 \quad (R = (2-x)/2) \end{cases}$$

$$V = \int_0^1 \frac{\pi}{2}\left(\frac{\sqrt{x}}{2}\right)^2 dx + \int_1^2 \frac{\pi}{2}\left(\frac{2-x}{2}\right)^2 dx$$

$$= \frac{\pi}{8}\int_0^1 x\,dx + \frac{\pi}{8}\int_1^2 4-4x+x^2\,dx$$

$$= \frac{\pi}{8}\left(\frac{x^2}{2}\Big|_0^1\right) + \frac{\pi}{8}\left(4x-2x^2+\frac{x^3}{3}\Big|_1^2\right)$$

$$= \frac{\pi}{8}\cdot\frac{1}{2} + \frac{\pi}{8}\left[\left(8-8+\frac{8}{3}\right) - \left(4-2+\frac{1}{3}\right)\right]$$

$$= \frac{\pi}{16} + \frac{\pi}{8}\left(\frac{8}{3} - \frac{7}{3}\right)$$

$$= \frac{\pi}{16} + \frac{\pi}{8}\cdot\frac{1}{3} = \frac{\pi}{16} + \frac{\pi}{24} = \frac{5}{48}\pi$$

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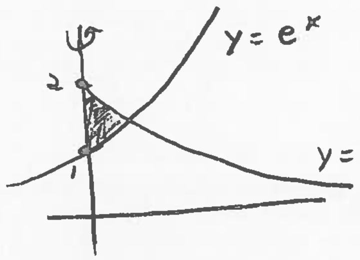


Use shells, (for y-axis rotation) $\int_a^b 2\pi r h \, dx$

$r = x$

$h = 2e^{-x} - e^{x}$

intersection:

$e^x = 2e^{-x}$

$e^{2x} = 2$

$2x = \ln(2)$

$x = \frac{1}{2}\ln(2)$

$y = \ln(\sqrt{2})$

$$V = \int_0^{\ln\sqrt{2}} 2e^{-x} - e^{x} \, dx$$

$$= -2e^{-x} - e^{x} \Big|_0^{\ln\sqrt{2}}$$

$$= \left(-2e^{-\ln\sqrt{2}} - e^{\ln\sqrt{2}}\right) - \left(-2e^{0} - e^{0}\right)$$

$$= \left(-2 \cdot \frac{1}{\sqrt{2}} - \sqrt{2}\right) - (-2-1)$$

$$= \left(-\sqrt{2} - \sqrt{2}\right) + 3$$

$$= \underline{\underline{3 - 2\sqrt{2}}} \approx 0.171573$$

* [This is the volume when the region is rotated about the y-axis. For x-axis, see next page.]

36 intersection:

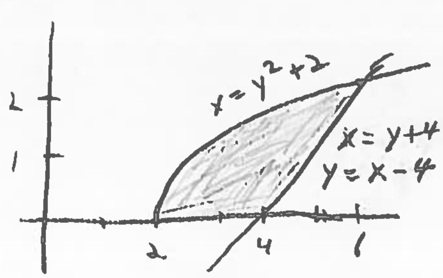
$y^2 + 2 = y - 4$

$y^2 - y + 6 = 0$

$(y+3)(y-2) = 0$

$y = -3$ or $\boxed{y = 2}$

intersection is at $y = 2$



a) $\int_0^2 (y+4) - (y^2+2) \, dy$

b) $V = \int_0^2 2\pi r h \, dy$ [shell method]

$$= \int_0^2 2\pi y \left((y+4) - (y^2+2)\right) dy$$

c) $V = \int_0^2 \pi (R^2 - r^2) \, dy$ [washers]

$$= \int_0^2 \pi \left((y+4)^2 - (y^2+2)^2\right) dy$$

d) $\int_0^2 A(y) \, dy$

$$= \int_0^2 \frac{\pi r^2}{2} \, dy \quad \text{[semicircle of radius r]}$$

$$= \int_0^2 \frac{\pi}{2} \left(\frac{(y+4) - (y^2+2)}{2}\right)^2 dy$$

since $r = \frac{(y+4) - (y^2+2)}{2}$

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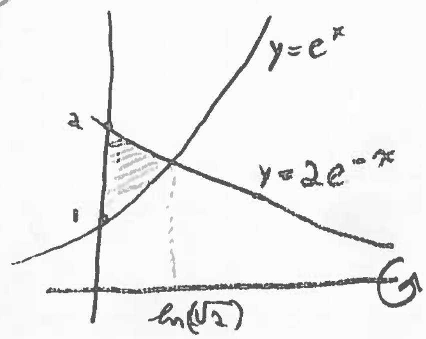

intersection:

$$e^x = 2e^{-x}$$
$$e^{2x} = 2$$
$$2x = \ln(2)$$
$$x = \tfrac{1}{2}\ln(2)$$
$$x = \ln(2^{1/2})$$
$$x = \ln(\sqrt{2})$$

Use washers for rotation about the x-axis

$$V = \int_0^{\ln\sqrt{2}} \pi(R_1^2 - R_2^2)\,dx$$

$$R_1 = 2e^{-x}, \quad R_2 = e^x$$

$$V = \int_0^{\ln\sqrt{2}} \pi(2e^{-x})^2 - \pi(e^x)^2\,dx$$

$$= \int_0^{\ln\sqrt{2}} 4\pi e^{-2x} - \pi e^{2x}\,dx$$

$$= 4\pi\cdot\left(-\tfrac{1}{2}e^{-2x}\right) - \pi\cdot\tfrac{1}{2}e^{2x}\Big|_0^{\ln\sqrt{2}}$$

$$= -2\pi e^{-2x} - \pi\frac{e^{2x}}{2}\Big|_0^{\ln\sqrt{2}}$$

$$= -2\pi e^{-2\ln\sqrt{2}} - \frac{\pi}{2}e^{2\ln\sqrt{2}} + 2\pi\cdot 1 + \frac{\pi}{2}$$

$$= \pi\left(\frac{5}{2} - 2e^{-\ln 2} - \frac{1}{2}e^{\ln 2}\right)$$

$$= \pi\left(\frac{5}{2} - 2\cdot\frac{1}{2} - \frac{1}{2}\cdot 2\right)$$

$$= \pi\left(\frac{5}{2} - 1 - 1\right)$$

$$= \pi\left(\frac{1}{2}\right)$$

$$= \frac{\pi}{2}$$