Section 8.2

(10) $\lim_{n\to\infty} \frac{n^{12}}{3n^{12}+4} = \frac{1}{3}$, since for the rational function $\frac{x^{12}}{3x^{12}+4}$,

$$\lim_{x\to\infty} \frac{x^{12}}{3x^{12}+4} = \lim_{x\to\infty} \frac{x^{12}}{3x^{12}} = \lim_{x\to\infty} \frac{1}{3} = \frac{1}{3}$$

("highest power rule")

or $$\lim_{x\to\infty} \frac{x^{12}}{3x^{12}+4} = \lim_{x\to\infty} \frac{12x^{11}}{3\cdot 12x^{11}} = \lim_{x\to\infty} \frac{1}{3} = \frac{1}{3}$$

(L'Hôpital's rule)

(12) $\lim_{n\to\infty} \frac{2e^n+1}{e^n} = 2$, since by L'Hôpital's rule,

$$\lim_{x\to\infty} \frac{2e^x+1}{e^x} = \lim_{x\to\infty} \frac{2e^x}{e^x} = \lim_{x\to\infty} 2 = 2$$

(34) $\lim_{n\to\infty} \frac{(-1)^{n+1}n^2}{2n^3+n} = 0$, since $\lim_{n\to\infty} \frac{n^2}{2n^3+n} = 0$ [Problem 4 on Lab 12]

or $\lim_{x\to\infty} \frac{x^2}{2x^3+x} = \lim_{x\to\infty} \frac{1}{2x} = 0$ (by the "highest power rule"), which means that both the positive and the negative terms in the series approach 0, Since all terms approach 0, the limit of the sequence is 0

(50) $\lim_{n\to\infty} 2^{n+1}3^{-n} = \lim_{n\to\infty} \frac{2\cdot 2^n}{3^n} = \lim_{n\to\infty} 2\cdot\left(\frac{2}{3}\right)^n = 0$

since this is a geometric sequence with $r = \frac{2}{3}$, and a geometric sequence converges to 0 if $|r| < 1$.

Section 8.3

(28) $\sum_{k=3}^{\infty} \frac{3\cdot 4^k}{7^k} = \sum_{k=3}^{\infty} 3\cdot\left(\frac{4}{7}\right)^k$ This is a geometric series

with $r = \frac{4}{7}$ and $a = \frac{3\cdot 4^3}{7^3}$, so it converges to

$$\frac{a}{1-r} = \frac{\frac{3\cdot 4^3}{7^3}}{1-\frac{4}{7}} = \frac{\frac{3\cdot 4^3}{7^3}}{\frac{3}{7}} = \frac{3\cdot 4^3}{7^3}\cdot\frac{7}{3} = \frac{4^3}{7^2} = \underline{\underline{\frac{64}{49}}}$$

(40) $\sum_{k=1}^{\infty} 3\left(-\frac{1}{8}\right)^{3k} = \sum_{k=1}^{\infty} 3\cdot\left(-\frac{1}{8^3}\right)^k$, a geometric series

with $r = -\frac{1}{8^3}$ and $a = -\frac{3}{8^3}$, which converges to

$$\frac{a}{1-r} = \frac{-\cancel{3}/8^3}{1-\left(-\frac{1}{8^3}\right)} = \frac{-3/\cancel{8^3}}{1+\frac{1}{8^3}} = \frac{-3}{8^3+1} = \underline{\underline{-\frac{3}{513}}} = -\frac{1}{171}$$

(46) $0.\overline{27} = \frac{27}{100} + \frac{27}{(100)^2} + \frac{27}{(100)^3} + \frac{27}{(100)^4} + \cdots$ (geometric series, $r = \frac{1}{100}$, $a = \frac{27}{100}$)

$$= \frac{\frac{27}{100}}{1-\frac{1}{100}} = \frac{27}{100-1} = \frac{27}{99} = \underline{\underline{\frac{3}{11}}}$$

[Could also be done directly: Let $x = 0.27272727\ldots$ Then $100x = 27.27272727\ldots$, so $100x - x = 27$, $99x = 27$, and $x = \frac{27}{99} = \frac{3}{11}$.]

(56) $\sum_{k=1}^{\infty}\left(\frac{1}{k+2} - \frac{1}{k+3}\right) = \left(\frac{1}{3}-\frac{1}{4}\right) + \left(\frac{1}{4}-\frac{1}{5}\right) + \left(\frac{1}{5}-\frac{1}{6}\right) + \left(\frac{1}{6}-\frac{1}{7}\right)\cdots$

The n-th partial sum is

$$S_n = \left(\frac{1}{3}-\frac{1}{4}\right) + \left(\frac{1}{4}-\frac{1}{5}\right) + \cdots + \left(\frac{1}{n+2}-\frac{1}{n+3}\right)$$
$$= \frac{1}{3} - \frac{1}{n+3}$$

So $\sum_{k=1}^{\infty}\left(\frac{1}{k+2}-\frac{1}{k+3}\right) = \lim_{n\to\infty} S_n = \lim_{n\to\infty}\left(\frac{1}{3}-\frac{1}{n+3}\right) = \frac{1}{3} - 0 = \underline{\underline{\frac{1}{3}}}$