MATH 131-01, Spring 2017 **Lab #1, January 17**

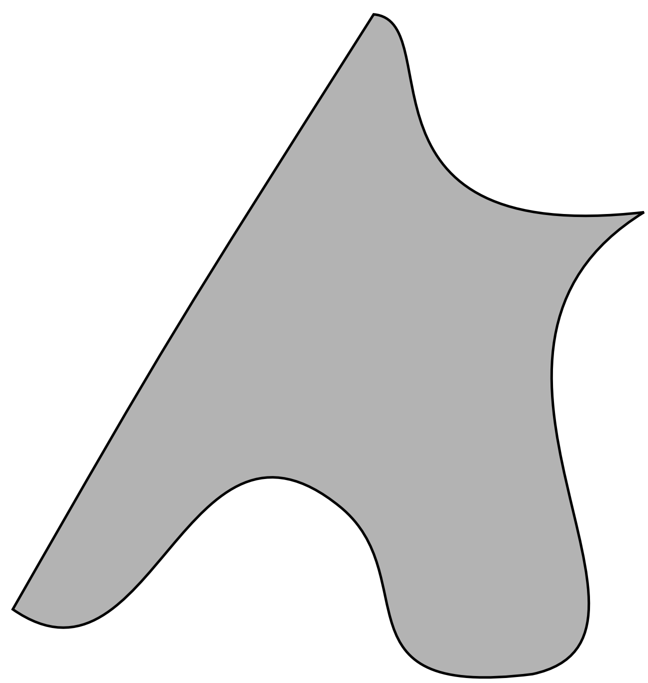
Welcome to the first lab of the semester! For the labs in this course, you will work in a group of 3 students (or 4 students ***only*** *if the number of students is not divisible by 3). You might need to meet outside of class to complete the lab. Your group will turn in a single lab report for the group. Answers must be clearly presented in full English sentences and correct mathematical notation. No credit is given for unsupported answers, and grades will be based partly on presentation. If you do not have a complete or final answer, you should write about attempts that you made, ideas that you had, partial solutions, etc. You are required to work together and discuss the problems. Everyone in the group should understand the solutions that you turn in, and everyone in the group is responsible for making sure that everyone else understands. Your group's report for this lab is due in class on Friday.*

1. The course handout says that the sum of the "infinite series"

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots + \frac{1}{2^n} + \cdots$$

is 2. What do you think? Does this make sense? Is it true? What does it have to do with the idea of *limits*? Hint: Find the difference between 2 and the finite sequence $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^n}$ for $n = 1$, $n = 2$, $n = 3$, and so on, until you see the pattern.

2. Suppose that you want to measure the area of the irregular region shown at the right.

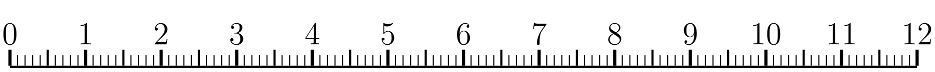


a) How would you go about finding an approximate value for the area? What process could you use? How could you increase the accuracy of your approximation? Try to come up with two different ways to get an approximation.

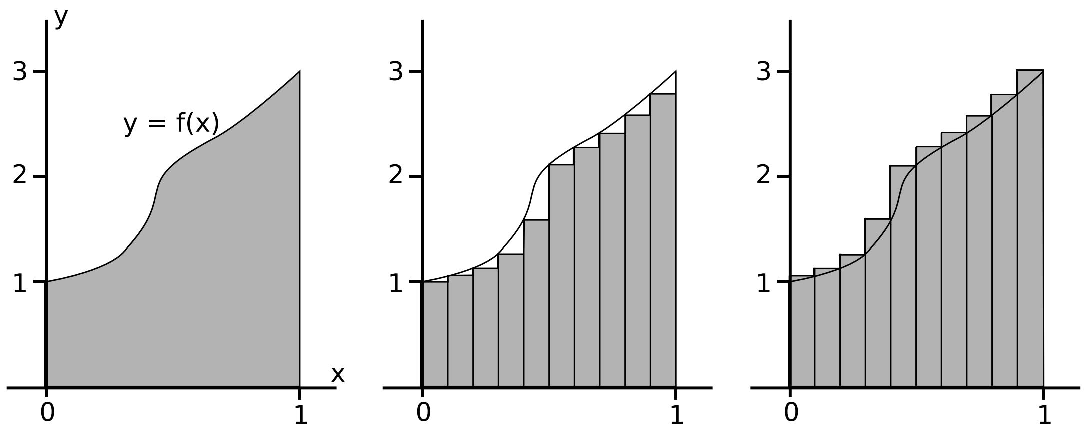
b) Find an approximate value for the area in square centimeters, using one of the techniques you described in part **a)**. If you don't have a centimeter ruler, you can use the one at the bottom of this page to measure distances. (One member of your group can fold the page along the edge of the ruler to make the measuring device.)

c) Assuming that you could measure very small distances accurately, how could you increase the accuracy of your approximation?

d) What does this have to do with *limits*?



3. The illustration below shows the graph of a function $y = f(x)$. $f(x)$ is continuous and increasing on the interval $[0, 1]$, and it satisfies $f(0) = 1$ and $f(1) = 3$. We are interested in the *area under the curve*, which is shown in gray in the graph on the left. The middle graph and the one on the right show two different ways of approximating the area of that region.



a) Consider the shaded area in the graph on the **right**. It is made up of ten rectangles, and the width of each rectangle is $\frac{1}{10}$. The height of each rectangle is given by the value of $f(x)$ at the **right** edge of the rectangle. Find a formula for the area of the shaded region, using values like $f(0.3)$ and $f(0.7)$. This is one possible estimate for the area under the curve. It is clear that this estimate is **larger** than the actual value.

b) Consider the shaded area in the **middle** graph. It is made up of ten rectangles, and the width of each rectangle is $\frac{1}{10}$. The height of each rectangle is given by the value of $f(x)$ at the **left** edge of the rectangle. Find a formula for the area of the shaded region, using values like $f(0.3)$ and $f(0.7)$. This is one possible estimate for the area under the curve. It is clear that this estimate is **smaller** than the actual value.

c) Find a formula for the **difference** between the shaded area in the graph on the right minus the shaded area in the middle graph. Then find the exact numerical value of the difference, given that $f(0) = 1$ and $f(1) = 3$.

d) Suppose that we were to repeat this exercise, but that we use **one hundred** rectangles instead of ten. We would still get two estimates for the area under the curve, one larger than the actual value and one smaller. What would be the **difference** between the two estimates, when 100 rectangles are used?