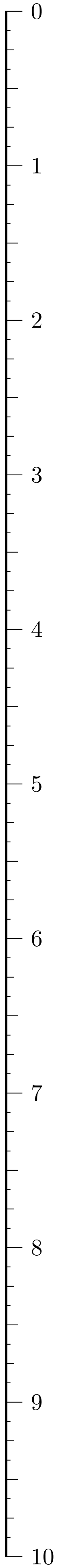

MATH 131-03, Spring 2017 **Lab #6, February 21**

1. The equation $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$ defines an ellipse (oval) that is centered at $(0,0)$ and stretches from $-a$ to a along the x-axis and from $-b$ to b along the y-axis. Solve the equation for y. How can the area of the ellipse be represented as the area between two curves? Write the integral that represents the area of the ellipse. Finally, use the substitution $u = \frac{x}{a}$ and the formula for the area of a circle to prove that the area of the ellipse is πab. How does this formula for the area of an ellipse relate to the formula for the area of a circle of radius r? (A circle is just a special kind of ellipse.) Your answer should include explanation and justification, written out in English sentences.

2. A "wedding bell" decoration will be available to pass around in the lab. The bell is a solid of rotation made by rotating the region shown on the back of this sheet about its long, straight side. Estimate the volume of the wedding bell, using ideas from the textbook about Riemann sums and volumes of rotation. You can use the ruler on the left to make the necessary measurements. (One member of the group can make a usable ruler by creasing a lab sheet along the line of the ruler.) Turn in a copy of the figure, with whatever marks and drawing you have made on it. Explain your work in words. You should come up with an actual numerical estimate of the area, measured in cubic inches.

3. (Gabriel's Horn) This problem concerns a famous example that is sometimes considered to be a mathematical paradox. Consider the infinite region bounded by the curve $y = \frac{1}{x}$, the x-axis, and the line $x = 1$. In a previous lab, you showed that this area is *infinite* by computing

$$\lim_{b\to\infty}\left(\int_1^b \frac{1}{x}\,dx\right) = \lim_{b\to\infty}\big(\ln(b)\big) = \infty$$

Suppose that this infinite region is rotated about the x-axis, producing a solid that has a shape like the horn of a trumpet, but infinitely extended.

a) Consider just the finite region bounded by $y = \frac{1}{x}$, $x = 1$, $x = b$, and the x-axis, for some number $b > 1$. Suppose that this region is rotated about the x-axis. Find the volume of the solid that is generated. This volume is a finite piece of the infinite horn. *Try to draw a reasonable figure of the solid.*

b) Take the limit as $b \to \infty$ of the volume that you found in part **1)**. This gives the volume of the entire infinite "horn." Note that the volume is *finite*.

c) Now the paradox: Imagine the horn as a shell filled with paint. Since the volume of the horn is finite, it can be filled with a finite amount of paint. Now think of the region that is rotated to generate the horn. This region is infinite, and therefore cannot be painted with a finite amount of paint. But *that region is actually **inside** the horn*! So, if the finite volume of the horn is filled with paint, then the infinite region has been covered with paint. Doesn't that mean that we have managed to paint an infinite region with a finite amount of paint? This seems to be a paradox. How can an infinite region fit inside a finite volume? Discuss this paradox, and try to find a way to resolve it. Write a report of your discussions, and of any ideas your have for resolving the paradox. (Hint: the resolution of the paint paradox is physical, not mathematical.)

4. There is no formula for the antiderivative of e^{-x^2}, but suppose that the region under the curve $y = e^{-x^2}$ on the interval $[0,\infty]$ is rotated about the y-axis to produce an "infinite hat." Consider the (finite) region under the curve $y = e^{-x^2}$ on the interval $[0,b]$, for some $b > 0$. Use the shell method to find the volume of the solid that is generated when this region is rotated about the y-axis. Now, take the limit as $b \to \infty$ to find the volume of the entire infinite hat.