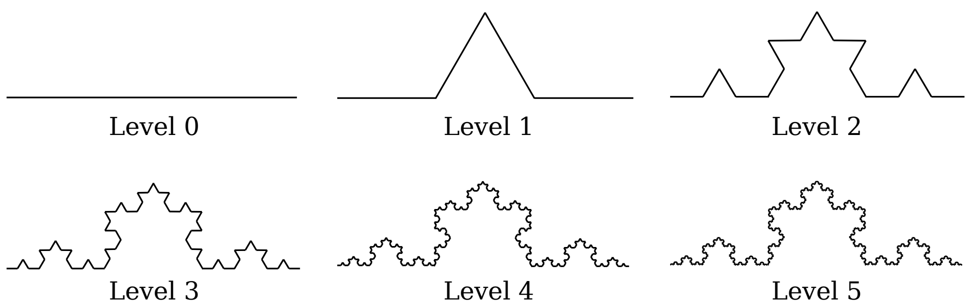
This lab is due at the start of next week's lab.

1. We have a formula for the length of a curve, if the curve is of the form $y = f(x)$ for some *differentiable* function of x. What about other curves? This problem investigates the famous *Koch Curve*. This curve is defined as a limit of a sequence of simpler curves. The first curve in the sequence is simply a straight line (Level 0 in the picture below). The next curve is obtained by removing the middle third of the Level 0 curve and replacing it with two straight lines. The Level 1 curve is made up of four straight line segments, all of the same length. Then it gets more interesting. The Level 2 curve is obtained by removing the middle third of each of the four line segments in the Level 1 curve and replacing it with a pair of line segments. There is another way of looking at the Level 2 curve: It is made up of four pieces, each a copy of the Level 1 curve but reduced to 1/3 of its original size. Similarly, the Level 3 curve is made up of four copies of the Level 2 curve, each 1/3 the size of the Level 2 curve. We can continue in this way indefinitely. Here are pictures of the first six levels:



If we continue this process forever, we get a "Level ∞" curve (which actually doesn't look a whole lot different from the Level 5 curve). That limiting curve is the Koch curve.

a) How many individual line segments are there in each of the six curves shown? Find a formula for the number of individual line segments in the Level N curve for any non-negative integer N? (Your answer should be a formula involving N.) Explain your work!

b) A level N curve is just a sequence of line segments, and all the line segments in a Level N curve are the same length. Let's assume that the length of the Level 0 curve is 1. What is the length of one of the individual line segments in a Level N curve? Why? (Your answer should be a formula involving N.)

c) The length of an entire Level N curve is just the sum of the lengths of all the individual line segments. Based on your answers from parts **a)** and **b)**, what is the total length of all the line segments in a Level 1 curve? In a Level 2 curve? Level 3? Find a formula for the length of a Level N curve, in terms of N.

d) What do you think about the Koch curve? Is it made up of line segments? What is its length? Does this whole idea make sense? Discuss the Koch curve and write a short essay about your ideas and conclusions.

(The Koch curve is an example of a *fractal*. It is "self-similar." In fact it is made up of four 1/3 sized exact copies of *itself*. It has a "fractal dimension" of $\ln(4)/\ln(3)$.)

2. The **centroid** of a finite region in the plane is the center point of the region in the sense that if you were to cut a copy of the region out of a sheet of material with uniform density, then the centroid would be the center of mass of that region—the point at which the region would exactly balance on the tip of your finger. If the region extends from a to b in the x-direction, and from c to d in the y-direction, then the x- and y-coordinates of the centroid are given by the formulas

$$x_c = \frac{\int_a^b x \cdot h(x)\,dx}{A} \qquad\qquad y_c = \frac{\int_c^d y \cdot w(y)\,dy}{A}$$

where A is the area of the region, $h(x)$ is the length of the vertical line through the region at x and $w(y)$ is the length of the horizontal line through the region at y.

The **Theorem of Pappus** says the when a region is rotated about a line that does not intersect the region, then the volume of the solid that is generated is equal to the area of the region multiplied by the distance traveled by the centroid as it rotates about the line.

a) Consider the region bounded by $y = x^2$ and the line $y = 9$. Find the centroid of this region.

b) Find the volume that is generated when this region is rotated about the line $y = 9$, using the method of disks.

c) Compute the same volume using the Theorem of Pappus, and check that the answer is the same as the value found in part **b)**.

d) Now, suppose the the region is rotated about the line $x = 4$. Use the method of shells to compute the value of the resulting solid.

e) Compute the same volume using the Theorem of Pappus, and check that the answer is the same as the value found in part **b)**.

f) The centroid of a disk is at the center of the disk. Suppose that that a disk of radius r is rotated about a line that is R units from the center of the disk, where $R > r$. The resulting doughnut-shaped solid is called a "torus." In class, we found the volume of the torus for the case $R = 3$ and $r = 1$. Use the Theorem of Pappus to find the volume of the torus in the general case, with an answer expressed in terms of r and R. Draw a sketch to show the disk and the axis of rotation, labeling R and r. And try to add to the sketch the path followed by the centroid of the disk.