This lab is due at the start of the next lab, in two weeks.

1. In class, we computed $\int e^{ax}\sin(x)\,dx$. We used integration by parts twice and got an equation that we could solve for the desired integral. Apply the same technique to compute

$$\int \sin(\ln(x))\,dx$$

2. In class, we derived a *reduction formula* for $\int x^n e^{ax}\,dx$. Apply integration by parts to the integral

$$\int (\ln(x))^n\,dx$$

to get a reduction formula and then use that reduction formula to find

$$\int (\ln(x))^3\,dx$$

3. Each of the following two problems requires *both* integration by parts and a substitution. Compute each integral. Start with a substitution. (To avoid confusion, use z or θ, not u, as the variable in the substitution.)

a) $\displaystyle\int \frac{\ln(\ln(x))}{x}\,dx$ **b)** $\displaystyle\int \cos(3\sqrt{x})\,dx$

4. Assume that a and b are constants and that $a \neq b$. Use algebra to verify that

$$\frac{1}{(x+a)(x+b)} = \frac{A}{x+a} - \frac{A}{x+b} \quad \text{where } A = \frac{1}{b-a}$$

and then use this fact to find the following integrals:

a) $\displaystyle\int \frac{1}{x^2+5x+6}\,dx$ **b)** $\displaystyle\int \frac{1}{z^2-1}\,dx$ **c)** $\displaystyle\int \frac{e^x}{e^{2x}+2e^x-8}\,dx$

For integral **c)**, start by applying a substitution!

Turn over for problem 5.

5. It is possible to use the following identity to compute certain integrals:

$$\sin(mx)\sin(nx) = \frac{1}{2}\big(\cos((m-n)x) - \cos((m+n)x)\big)$$

Note that if $m = n$, this becomes:

$$\sin^2(nx) = \frac{1}{2}\big(1 - \cos(2nx)\big)$$

a) Use the first identity to compute the indefinite integral $\int \sin(12x)\sin(7x)\,dx$

b) Use these identities to show that $\frac{2}{\pi}\int_0^\pi \sin(mx)\sin(nx)\,dx$ is 0 when $n \neq m$ and that it is 1 when $n = m$, where n and m are positive integers.

c) Suppose that $f(x)$ is a function that has the form $f(x) = \displaystyle\sum_{n=1}^{K} a_n \sin(nx)$ for some positive integer K and some constants $a_1, a_2, \ldots, a_K$. Suppose that we know that $f(x)$ can be written in this form, but we don't know the constants a_n. Use the properties of integrals and the results from part **a)** to show that a_n can be computed as $a_n = \frac{2}{\pi}\int_0^\pi f(x)\sin(nx)\,dx$. This shows that, starting from the function without knowing the values of the constants a_n, we can use integration to discover their values.

(The function $\sin(nx)$ represents a "pure tone" or a "pure frequency" of n. $f(x)$ is a combination of these pure tones. This exercise shows how $f(x)$ can be analyzed to determine how much of each pure frequency it contains. This applies, for example, to a sound that is made up of a finite number of pure tones. This idea is the beginning of *digital signal analysis*, which is used in computer processing of sound waves and other signals. Extended to infinite sums, it leads to *Fourier analysis*, one of the core fields of applied mathematics.)