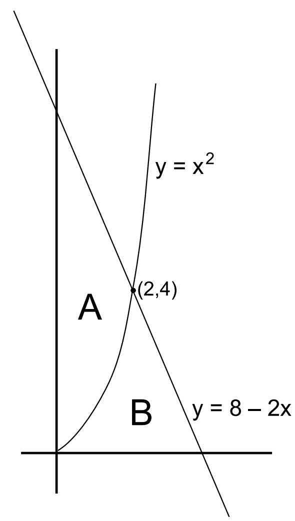
This lab is review for tomorrow's test. It will not be collected. Answers will be available before the end of the lab. Almost all of the problems come directly from tests given in previous semesters.

1. The region under the curve $y = x^2 + 1$, for $0 \le x \le 2$ is rotated about the x-axis. Use the "disk method" to find the volume of the resulting solid.

2. The region under the curve $y = x^2 + 1$, for $0 \le x \le 2$ is rotated about the y-axis. Use the "shell method" to find the volume of the resulting solid.

3. The x-axis, the y-axis, the line $y = 8 - 2x$, and the curve $y = x^2$ bound two finite regions, A and B, in the first quadrant, as shown. Find the area of A and find the area of B. (Note that you can do this with just **one** integral, plus some geometry.)



4. Suppose that $f(x)$ is a differentiable function on the interval $[a, b]$.

a) Give the integral formula for the *arc length* of the curve $y = f(x)$ for $a \le x \le b$.

b) *Use the integral formula* to explain why the arc length must be greater than or equal to $b - a$.

c) For which functions $y = f(x)$ will the arc length have the minimum possible value, $b - a$? Why? Explain your answer *using the integral formula for arc length.*

5. Apply the arc length formula to set up an integral for the length of the curve $y = \sin(x)$, for $0 \le x \le \pi$. **Do not** evaluate the integral.

6. Consider the region bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{1}{2}x$. (These graphs intersect at $(4, 2)$.)

a) Set up but **do not evaluate** an integral for the volume of the solid that is generated when this region is rotated about the x-axis, using the "method of washers."

b) Set up but **do not evaluate** an integral for the volume of the solid that is generated when this region is rotated about the x-axis, using the "method of shells."

c) Set up but **do not evaluate** an integral for the volume of the solid that is generated when this region is rotated about the y-axis (using any method).

d) Set up but **do not evaluate** an integral for the volume of the solid that is generated when this region is rotated about the line $y = -3$ (using any method).

7. The base of a solid is the region under the curve $y = 1 - x^2$ for $-1 \le x \le 1$. Cross sections perpendicular to the x-axis are semicircles. Write down an integral that gives the volume of the solid. Do not evaluate the integral.

8. Suppose that $f(x) > 0$ on the interval $[a, b]$, and consider the solid that is generated when the region under $y = f(x)$, for $a \le x \le b$, is rotated about the y-axis. Suppose that the interval is divided into n equal subintervals, and that for each k, x_k^* is some point in the i-th subinterval. Explain *why* the Riemann Sum

$$\sum_{k=1}^{n} 2\pi x_k^* f(x_k^*)\,\Delta x$$

is a reasonable approximation for the volume of the solid, and explain how this leads to the integral formula for the "shell method." Illustrate your answer with at least one picture.

9. Use *integration by parts* to find $\displaystyle\int \ln(x)\,dx$

10. Use *integration by parts* to find $\displaystyle\int x\tan^{-1}(x)\,dx$

11. Use the *method of partial fractions* to find $\displaystyle\int \frac{x+2}{(x-1)(2x+1)}\,dx$

12. Show the form of the *partial fractions decomposition* of the following rational function. **Do not** evaluate the constants A, B, ...!

$$\frac{2x^2+7}{x(x^2+4)(2x+7)}$$

13. Apply integration by parts to find a *reduction formula* for $\displaystyle\int x^n e^x\,dx$.

14. Compute each of the following indefinite integrals, using any technique:

a) $\displaystyle\int x\cos(x^2)\,dx$

b) $\displaystyle\int x\cos(x)\,dx$

c) $\displaystyle\int e^{\sqrt{x}}\,dx$

d) $\displaystyle\int \frac{2x^2+5}{x^2+1}\,dx$

e) $\displaystyle\int \sin^4(t)\cos^3(t)\,dt$ (Hint: Use $\cos^2(t) = 1 - \sin^2(t)$, followed by substitution.)