Math 131-03, Spring 2017 — Third Test Info

The third test for this course will be given in class on Wednesday, April 19. The test covers Section 7.8 through 8.4. This includes improper integrals, simple differential equations and initial value problems, basics of infinite sequences and series, geometric sequences and series, the Divergence test, and the Integral Test. It is possible that you will need L'Hôpital's rule, in its basic form, for computing a limit. It is possible that you might have to compute an integral in order to apply the Integral test.

As usual, you will not need a calculator for the test. A basic non-graphing calculator will be provided to you, and you will be permitted to use only the calculator that is provided. Scrap paper will also be provided. All you need is a pencil or pen.

Here are some terms and ideas that you should be familiar with for the test:

differential equation

what it means to be a solution of a differential equation

separable differential equations and how to solve them

initial value problem

solving initial value problems

improper integrals on infinite domains: $\displaystyle\int_a^\infty f(x)\,dx$, $\displaystyle\int_{-\infty}^b f(x)\,dx$, and $\displaystyle\int_{-\infty}^\infty f(x)\,dx$,

improper integrals of unbounded functions, such as $\displaystyle\int_0^1 \ln(x)\,dx$

why improper integrals are "improper"

infinite sequences: $a_o,\, a_1,\, a_2,\, \dots,\, a_n,\, \dots$

the notation $\{a_n\}_{n=0}^\infty$ (where the starting value can be anything, not just 0)

the limit of an infinite sequence, $\displaystyle\lim_{n\to\infty} a_n$

convergent sequence (limit exists, as a finite number)

divergent sequence (limit does not exist, including the case of an infinite limit)

properties of limits of sequences, such as $\displaystyle\lim_{k\to\infty}(a_k+b_k) = (\lim_{k\to\infty} a_k) + (\lim_{k\to\infty} b_k)$

an increasing sequence either converges or else it diverges to ∞

a bounded increasing sequence converges

geometric sequence, $\{ar^n\}_{n=0}^{\infty}$, where r is a constant and $a \neq 0$

$\{r^n\}_{n=1}^{\infty}$ converges to 0 if $-1 < r < 1$, converges to 1 if $r = 1$, and diverges otherwise

infinite series, $\displaystyle\sum_{n=0}^{\infty} a_n$ (where the lower limit can be anything, not just 0)

properties of series: $\displaystyle\sum_{n=1}^{\infty} c \cdot a_n = c \cdot \sum_{n=1}^{\infty} a_n$ and

$$\sum_{n=1}^{\infty}(a_n + b_n) = \left(\sum_{n=1}^{\infty} a_n\right) + \left(\sum_{n=1}^{\infty} a_n\right),$$ if the limits on the right exist

a series with positive terms must either converge or else it diverges to ∞

the sequence of partial sums for an infinite series, $s_n = \displaystyle\sum_{k=0}^{n} a_k$

the sum of an infinte series, $\displaystyle\sum_{n=0}^{\infty} a_n = \lim_{n\to\infty}\left(\sum_{k=0}^{n} a_k\right)$ (the limit of the sequence of partial sums)

geometric series, $\displaystyle\sum_{n=0}^{\infty} ar^n$, where r is constant and $a \neq 0$

a geometric series with $|r| \geq 0$ diverges

a geometric series with $|r| < 1$ converges to $\frac{a}{1-r}$

if $\displaystyle\sum_{n=0}^{\infty} a_n$ converges, then $\displaystyle\lim_{n\to\infty} a_n$ must converge to 0

the Divergence Test: If $\displaystyle\lim_{n\to 0} a_n \neq 0$, then $\displaystyle\sum_{n=0}^{\infty} a_n$ diverges

the Integral Test: If $f(x)$ is a positive, decreasing function on $[0, \infty)$, then

(1) If $\displaystyle\int_1^{\infty} f(x)\,dx$ converges, then $\displaystyle\sum_{n=0}^{\infty} a_n$ also converges

(2) If $\displaystyle\int_1^{\infty} f(x)\,dx$ diverges, then $\displaystyle\sum_{n=0}^{\infty} a_n$ also diverges

how to prove the Integral Test, using a picture

p-series: The series $\displaystyle\int_1^{\infty} \frac{1}{x^p}\,dx$ converges if $p > 1$ and diverges if $p \leq 1$

Using all of these tests to determine whether a given infinite sequence converges or diverges