Math 135, Fall 2019, Homework 1

This homework is due on Monday, September 2, in class. Some of the exercises are based on material that we will not cover in class until Friday. All exercises, without exception, require an explanation or justification in words! Remember that you can discuss the problems with other people in the class, but you should write up your own solutions.

Many of the exercises are from the textbook. When a problem is from the textbook, the section and problem number is given in parentheses. For example, (1.4.8) would be problem 8 from Section 1.4. The textbook has a section giving solutions to all of the odd numbered exercises. In many cases, you can get hints about an even-numbered exercise by looking at nearby odd-numbered exercises and their solutions.

Exercise 1 (1.1.18, 1.1.20, 1.1.26)**.** Write each of the following sets using set-builder notation:

- **a)** $\{0, 4, 16, 36, 64, 100, \dots\}$
- **b)** $\{\dots, -8, -3, 2, 7, 12, 17, \dots\}$
- **c)** $\{\dots, \frac{1}{27}, \frac{1}{9}, \frac{1}{3}, 1, 3, 9, 27 \dots\}$

Exercise 2 (1.2.4)**.** Write out the set by listing its elements between braces:

$$\{n \in \mathbb{Z} \colon 2 < n < 5\} \times \{n \in \mathbb{Z} \colon |n| = 5\}$$

Exercise 3 (1.4.16, 1.4.20)**.** Suppose that A and B are finite sets and that $|A| = n$ and $|B| = m$. Find the following cardinalities:

- **a)** $|\mathscr{P}(A) \times \mathscr{P}(B)|$
- **b)** $|\{X \colon X \in \mathscr{P}(A) \text{ and } |X| \leq 1\}|$

Exercise 4. Suppose that A and B are sets. Is it true that $\mathscr{P}(A) \subseteq \mathscr{P}(B)$? (Justify your answers!)

Exercise 5. Let U be some universal set, and suppose $A \subseteq U$. Consider the set $\overline{\overline{A}}$, the complement of the complement of A. What set is $\overline{\overline{A}}$ equal to? (This is, how can it be simplified? Justify your answer!)

Exercise 6 (1.8.4)**.** For each $n \in \mathbb{N}$, let $A_n = \{-2n, 0, 2n\}$. Compute the sets $\displaystyle\bigcup_{i \in \mathbb{N}} A_i$ and $\displaystyle\bigcap_{i \in \mathbb{N}} A_i$

Exercise 7 (1.8.6)**.** Compute the sets $\displaystyle\bigcup_{i \in \mathbb{N}} [0, i+1]$ and $\displaystyle\bigcap_{i \in \mathbb{N}} [0, i+1]$

Exercise 8 (1.8.12)**.** If $\displaystyle\bigcap_{\alpha \in I} A_\alpha = \bigcup_{\alpha \in I} A_\alpha$, what do you think can be said about the relationships among the sets A_α? (Explain!)

Exercise 9 (1.8.14)**.** If $J \neq \varnothing$ and $J \subseteq I$, is is true that $\displaystyle\bigcap_{\alpha \in I} A_\alpha \subseteq \bigcap_{\alpha \in J} A_\alpha$? (Explain!)

Exercise 10. Suppose that A, B, and C are sets and that $A \times C = B \times C$. It is not always true that $A = B$ in that case. Find an example where $A \times C = B \times C$ but $A \neq B$. Can you find a condition on C such that if C satisfies that condition, then $A \times C = B \times C$ will imply that $A = B$. Explain! (Hint: The cross product, $\times$, is similar in some ways to multiplication. What would happen if A, B, and C are numbers and $\times$ is ordinary multiplication?)