Math 135, Fall 2019, Final Exam Information

The final exam for Math 135 will take place in our regular classroom at the officially designated time: Saturday, December 14, at 1:30 PM. The format will be similar to the two in-class tests, except that essay questions can be longer and proofs can be less trivial. Some of the things that were proved in class have reasonably easy proofs that could be asked on the exam. In particular, you will be asked to give a proof of at least one of the following: the set of real numbers is uncountable, there is no bijective function from a set to its power set, congruence modulo n is an equivalence relation, the equivalence classes of an equivalence relation on a set form a partition of that set, a function has an inverse function if and only if it is bijective.

You should know all the proof techniques that we have covered, including direct proof, proof by contrapositive, proof by contradiction, proof by cases, proving if-and-only-if statements, and proof by mathematical induction. In particular, you will certainly be asked to do a proof by induction.

The exam will concentrate on material that has been covered since the second test, but it will also include questions on older material. You should certainly expect some questions on mathematical logic and sets. You are responsible for all of the material that has been covered in the course (except for topology and group theory). This includes the entire textbook, aside from Chapters 3 and 13 and a few special topics such as perfect numbers and Fibonacci numbers. You should review the study guides for the two in-class tests, which you can find on the course web page. You might also want to review homework assignments. My sample solutions can all be found on-line.

The test will be six to eight pages long. It can include definitions, shorter and longer essay-type questions, proofs, and more computational exercises. The exam period is three hours. I expect the test to take two hours or less to complete, but you can use the full three hours if you need it.

Some things that were covered since the second test:

relation on a set A (a subset of $A \times A$)

representing relations as graphs (diagrams)

properties of a relation: reflexive, symmetric, transitive, antisymmetric

the relation $\equiv \pmod{n}$ on $\mathbb{Z}$ is reflective, symmetric, and transitive

the relation $\leq$ on $\mathbb{R}$ is reflexive, transitive, and antisymmetric

equivalence relation (symmetric, reflexive, and transitive)

equivalence class $[x]$ of an element $x \in A$ under an equivalence relation on A

partition of a set

how partitions relate to equivalence relations

equivalence classes under the relation $\equiv \pmod{n}$ on $\mathbb{Z}$

the set $\mathbb{Z}_n = \{[0], [1], [2], \ldots, [n-1]\}$, and the binary relations $\oplus$ and $\odot$ on $\mathbb{Z}_n$

relation from a set A to a set B (a subset of $A \times B$)

function from a set A to a set B; the notation $f\colon A \to B$
domain, codomain, and range of a function
defining a function as set of ordered pairs
visualizing functions with diagrams
injective, surjective, and bijective functions
composition of functions, $f \circ g$
B^A, the set of functions from the set A to the set B
identity function on a set A, $i_A\colon A \to A$
inverse of a relation
inverse function
the inverse function, f^{-1}, for a function f satisfies $f^{-1}(f(x)) = x$ and $f(f^{-1}(y)) = y$
a function has an inverse function if and only if it is bijective
the image $f(X)$ of a subset $X \subseteq A$, for a function $f\colon A \to B$
the preimage $f^{-1}(Y)$ of a subset $Y \subseteq B$, for a function $f\colon A \to B$
infinite sets
two sets A and B have the same cardinality if there is a bijection from A to B
notation: $|A| = |B|$ or $A \approx B$ if and only of there is a bijection from A to B
countably infinite sets; examples: $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{N} \times \mathbb{N}$
a set is countably infinite if and only if its elements can be arranges in an infinite list
uncountable sets; examples: $\mathbb{R}$, $\mathscr{P}(\mathbb{N})$, $\mathscr{P}(\mathscr{P}(\mathbb{N}))$
Cantor's diagonal argument to prove that $\mathbb{R}$ is uncountable
there is no bijection from a set A to its power set $\mathscr{P}(A)$
notation: $|A| \leq |B|$ or $A \preccurlyeq B$ if and only if there is an injection from A to B
Cantor-Bernstein-Schröder Theorem: If $|A| \leq |B|$ and $|B| \leq |A|$, then $|A| = |B|$
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Some particularly important topics from earlier in the course:

logical operations (and, or, not, implies, if-and-only-if)
truth tables
translating between English and logic
quantifiers, $\forall$ and $\exists$
negating logical statements
proof techniques
integer properties: even and odd, prime numbers, $a \mid b$, $\gcd(a, b)$, $a \equiv b \pmod{n}$
rational and irrational numbers
sets, elements, subsets
set operations: union, intersection, set difference, power set, cross product