*This homework is due by **noon** on Tuesday, October 13,*

Problem 1. Find the rank of each matrix:

(a) $\begin{pmatrix} 1 & 3 & -2 & 4 \\ 2 & 1 & 1 & 3 \\ -1 & 2 & -3 & 1 \end{pmatrix}$ **(b)** $\begin{pmatrix} 1 & 0 & 3 & 5 & 2 \\ -1 & 2 & 1 & -2 & 3 \\ 2 & 4 & 0 & 1 & 1 \\ 3 & 4 & 3 & 6 & 3 \\ 1 & 6 & 1 & -1 & 4 \end{pmatrix}$

Problem 2. Suppose that $h\colon \mathbb{R}^3 \to \mathbb{R}^3$ is a homomorphism that satisfies

$$h\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}0\\2\\1\end{pmatrix}, \qquad h\begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}3\\-1\\0\end{pmatrix}, \quad \text{and} \quad h\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}1\\2\\3\end{pmatrix}$$

(a) Find $h\begin{pmatrix}-2\\3\\1\end{pmatrix}$. (Remember that h is a homomorphism.)

(b) For any vector $\begin{pmatrix}a\\b\\c\end{pmatrix} \in \mathbb{R}^3$, find $h\begin{pmatrix}a\\b\\c\end{pmatrix}$, writing the answer in terms of a, b, and c.

(c) Find a specific vector $\begin{pmatrix}a\\b\\c\end{pmatrix} \in \mathbb{R}^3$ such that $h\begin{pmatrix}a\\b\\c\end{pmatrix} = \begin{pmatrix}-1\\0\\2\end{pmatrix}$.

Problem 3. In class, we showed that the function from $\mathscr{P}_3$ to $\mathscr{P}_3$ that maps the polynomial $p(x)$ to the polynomial $p(x-1)$ is an automorphism of $\mathscr{P}_3$. Define the homomorphism $h\colon \mathscr{P}_2 \to \mathscr{P}_2$ by $h(p(x)) = p(2x+5)$. (You do not have to show that this function is a homomorphism. Note that it is defined on $\mathscr{P}_2$, not $\mathscr{P}_3$.)

(a) Show that h is bijective by finding an inverse function.

(b) Write out $h(a+bx+cx^2)$ as a polynomial in standard form $(d+ex+fx^2)$, where d, e, f are expressed in terms of a, b, c).

Problem 4. Define $f\colon \mathbb{R}^4 \to \mathbb{R}^4$ by $f\begin{pmatrix}a\\b\\c\\d\end{pmatrix} = \begin{pmatrix}d\\c\\b\\a\end{pmatrix}$. Show by direct calculation that f is a homomorphism, and show that it is in fact an automorphism by finding its inverse.

Problem 5. Suppose that V, W, and X are vector spaces and that $f\colon V \to W$ and $g\colon W \to X$ are homomoprhisms. Recall that the compostion, $g \circ f$, of g and f is defined to be the function from V to X given by $g \circ f(\vec{v} = g(f(\vec{v}))$ for $\vec{v} \in V$. Show that $g \circ f$ is a homomorphism. (This is easy! Just check the two conditions for a function to be a homomorphism.)