The first test for this course will be given in class on Friday, October 2. It covers all of the material from the course through September 25. This includes Chapters 1 and 2 up to Chapter 2 Section III.2.

For the test, you should know and understand all the definitions and theorems that we have covered. You should be able to work with matrices, systems of linear equations, and vector spaces. The test will include some "short essay" questions that ask you to define something, or discuss something, or explain something, and so on. Other than that, you can expect most of the questions to be similar to problems that have been given on the homework, but only problems that are not expected to take up too much time. You might be asked to do a few proofs, but they will be short and straightforward. I will ***not*** *ask you to give the full definition of vector space. You should know the standard notations for row operations and what they mean.*

Here are some terms and ideas that you should be familiar with for the test:

systems of linear equations

solution set of a linear system of equations

Gauss's method (row reduction)

the three row operations

notations for row operations: $k\rho_i + \rho_j$, $\rho_i \leftrightarrow \rho_j$, $k\rho_j$

applying row operations to a system of equations

echelon form for a system of linear equations

matrix; rows and columns of a matrix; $m \times n$ matrix

representing a system of linear equations as an augmented matrix

row operations and echelon form for matrices

vectors in $\mathbb{R}^n$

vector addition and scalar multiplication for vectors in $\mathbb{R}^n$

linear combination of vectors in $\mathbb{R}^n$

expressing the solution set of a linear system in vector form

homogeneous system of linear equations

associated homogeneous system for a system of linear equations

a solution set has the form $\{\vec{p}+\vec{h} \,|\, \vec{h} \text{ solves the homogeneous system}\}$; $\vec{p}$ is a particular solution

solution set of a homogeneous system has the form $\{a_1\vec{v}_1 + a_2\vec{v}_2 + \cdots a_k\vec{v}_k \,|\, a_1, a_2, \ldots, a_k \in \mathbb{R}\}$

a linear system of equations can have zero, one, or an infinite number of solutions

a homogeneous system can have one solution (namely $\vec{0}$) or an infinite number of solutions

leading variables in a system in echelon form

free variables

singular and non-singular matrices

dot product of vectors in $\mathbb{R}^n$: $\vec{u}\cdot\vec{v} = v_1u_1 + v_2u_2 + \cdots + v_nu_n$

length of a vector in $\mathbb{R}^n$: $|\vec{v}| = \sqrt{\vec{v}\cdot\vec{v}} = \sqrt{v_1^2 + v_2^2 + \cdots + v_n^2}$

angle between two vectors in $\mathbb{R}^n$: $\cos(\theta) = \frac{\vec{v}\cdot\vec{u}}{|\vec{v}||\vec{u}|}$

orthogonal vectors in $\mathbb{R}^n$, $\vec{v}\cdot\vec{u} = 0$

reduced echelon form

row equivalence of matrices

every matrix is row equivalent to a unique reduced echelon form matrix

linear combination lemma: a linear combination of linear combinations is a linear combination

vector space: closure, commutativity, associativity, zero vector, additive inverse, etc.

$\mathbb{R}^n$ as a vector space with the usual vector addition and scalar multiplication

the set of polynomials, $\mathscr{P}_n = \{p(x)\,|\,p \text{ is a polynomial of degree } \le n\}$, as a vector space

the set of all polynomials, $\mathscr{P}$, as a vector space

the set $\mathscr{F}$ of functions from $\mathbb{R}$ to $\mathbb{R}$, as a vector space

showing that a set with addition and scalar multiplication operations is or is not a vector space

subspace of a vector space

proving a subset is a subspace: non-empty and closed under addition and scalar multiplication

the solution set of a homogeneous system of linear equations in n variables is a subspace of $\mathbb{R}^n$

the span of a subset of vector space (the set of linear combinations of elements of the subset)

the span of a subset of a vector space is a subspace of that vector space

linearly dependent subset and linearly independent subset of a vector space

$\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_k$ are linearly independent iff $c_1\vec{v}_1 + c_2\vec{v}_2 + \cdots + c_k\vec{v}_k = \vec{0}$ implies $c_1 = c_2 = \cdots = c_k = 0$

the vectors $\vec{e}_1, \ldots, \vec{e}_n$ in $\mathbb{R}^n$ are linearly independent and span $\mathbb{R}^n$

$\langle \vec{e}_1, \ldots, \vec{e}_n \rangle$ is the standard basis of $\mathbb{R}^n$

using linear equations to test spanning, linear independence, etc.

a sequence $\langle \vec{b}_1, \vec{b}_2, \ldots, \vec{b}_n \rangle$ is a basis of V if and only if every $\vec{v} \in V$ can be written uniquely as a linear combination of $\vec{b}_1, \vec{b}_2, \ldots, \vec{b}_n$.

$\mathrm{Rep}_B(\vec{v})$, the representation of a vector $\vec{v}$ in a basis B

finite-dimensional vector space

dimension of a vector space

all bases of a finite-dimensional vector space have the same dimension