This homework is due on Friday, November 11. At that time, the second take-home test will be avaialble, and that test will be due at the start of class on Friday, November 18. The second in-class test will be given on November 18. Resubmissions for Homework 9 will be due on November 21. There might be one more homework assignment, which will be due on the last day of class. Note that this homework is worth a total of 32 points.

Problem 1. A previous homework problem showed that $\frac{1}{2}x|x|$ is an antiderivative for $|x|$. Using that fact, evaluate $\int_{-3}^{5} |x|\,dx$ **using the first Fundamental Theorem of Calculus once**. Explain why the answer makes sense geometrically (in terms of area).

Problem 2. **(a)** Suppose that $\sum_{k=1}^{\infty} a_k$ is a convergent series, and $\sum_{k=1}^{\infty} b_k$ is a divergent series. Show that $\sum_{k=1}^{\infty}(a_k + b_k)$ diverges. [Hint: Proof by contradiction will work.]

(b) Suppose that $\sum_{k=1}^{\infty} a_k$ and $\sum_{k=1}^{\infty} b_k$ are both divergent. Give a simple example to show that $\sum_{k=1}^{\infty}(a_k + b_k)$ does not necessarily diverge.

Problem 3. A nonnegative series must either converge (absolutely) or diverge to $+\infty$. Classify each of the following nonnegative series as either convergent or divergent. In all cases, explain your reasoning, being explicit about any convergence tests that you apply.

a) $\displaystyle\sum_{k=1}^{\infty} \frac{3k^2}{7k^5 + 2k^2}$ **b)** $\displaystyle\sum_{k=1}^{\infty} \frac{k}{\sqrt{k^2+1}}$ **c)** $\displaystyle\sum_{n=0}^{\infty} \pi^{-n}$ **d)** $\displaystyle\sum_{n=1}^{\infty} \frac{2^n + 3^n}{4^n}$

e) $\displaystyle\sum_{m=1}^{\infty} \frac{1+\sin(m)}{5^m}$ **f)** $\displaystyle\sum_{n=1}^{\infty} \frac{n^6}{5^n}$ **g)** $\displaystyle\sum_{k=1}^{\infty} \frac{(k!)^2}{(2k)!}$ **h)** $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n}}$

Problem 4. A series of positive and negative terms can either diverge, converge absolutely, or converge conditionally. Classify each of the following series as one of divergent, absolutely convergent, or conditionally convergent. In all cases, explain your reasoning, being explicit about any convergence tests that you apply.

a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{3^n}$ **b)** $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k+1}}$

c) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n \ln(n)}{n}$ **d)** $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k}{k^k}$

Problem 5. The series $\sum_{k=2}^{\infty} \frac{1}{k\ln(k)}$ diverges, but none of the tests that we have covered can prove it. Note that $\lim_{n\to\infty}\left(\int_2^n \frac{1}{x\ln(x)}dx\right) = \lim_{n\to\infty}\big(\ln(\ln(x)) - \ln(\ln(2))\big) = +\infty$. Also note that $f(x) = \frac{1}{x\ln(x)}$ is decreasing. [You do not have to prove these facts.] Show that the partial sum, $s_n = \sum_{k=2}^{n} \frac{1}{k\ln(k)}$, satisfies

$$s_n \geq \int_2^{n+1} \frac{1}{x\ln(x)}\,dx$$

by considering the upper sum using the partition $\{2, 3, 4, \ldots, n+1\}$ of the interval $[2, n+1]$, and conclude that $\sum_{k=2}^{\infty} \frac{1}{k\ln(k)}$ diverges. (Note that this example is a special case of something called the "integral test.")